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Boundary Element Method Calculation of Moment Transfer Parameters in Slab-Column Connections

by Mohammed A. Nazief, Youssef F. Rashed, and Wael M. El-Degwy

In this paper, a new method is suggested to compute the values of γ_v and γ_f parameters at any slab-column connection. The method is based on an elastic analysis of the overall slab using the boundary element method (BEM) via the shear deformable plate bending theory. The stress resultants along the critical section perimeter are computed in a semi-analytical, semi-numerical manner. This method is implemented into a computer program to help analyze several columns with several critical section positions. Values obtained using the present analysis are compared against those given by the ACI 318 (2008) and by previous research based on the finite element method. It is recommended to use the present method, especially for a column having irregular shapes or having a large rectangularity ratio.

Keywords: boundary element method; flat plates; moment transfer parameters; punching shear; slab-column connection.

INTRODUCTION

Punching analysis of slab-column connections is an important problem in structural engineering. The importance of such problem increases when having unbalanced moment transfer due to lateral forces. In practice, engineers usually uses the code (for example, the ACI 318[1]) to compute the so-called moment transfer parameters γ_v (percentage of moment transferred by shear forces) and γ_f (percentage of moment transferred by bending and twisting moments). As will be seen in this paper, however, sometimes such values are not accurate, especially when dealing with columns with large rectangularity ratio.

Several researches have considered the analysis of such connection. Some of these researches are based on experimental investigations such as the work of Kinnunen and Naylander,[2] Broms,[3] and Teng et al.[4] Other research is based on the analytical methods of Mast[5] and Elgabry and Ghali.[6] It has to be noted that in both experimental and analytical methods, many simplifications are introduced to allow carrying out the desired experimental work or to be able to study forces at relevant connections. In all cases, without exception, usually a simple slab or single slab-column connection was studied.

Elgabry and Ghali[6] used a numerical method to compute moment transfer parameter by performing finite element analysis (FEM) of a simple slab. The slab is modeled using the shear-deformable plate bending theory[7] and the columns are modeled as boundary conditions. Unbalanced moments are introduced at the column face, using equivalent deflections and rotations introduced at the finite element nodes. They studied several cases including interior, edge, and corner columns with a different rectangularity ratio. Several charts were plotted against the values given by the ACI equations. They concluded that the values given by the ACI equations should generally not be used for all column positions. It has to be noted that, despite the generality of the numerical model given by Elgabry and Ghali,[6] it is not practical and cannot be used in practice. Moreover, using finite elements in places having stress concentrations (such as the slab-column connections) usually loses accuracy. More comments regarding the Elgabry and Ghali[6] model will be discussed in this paper.

The boundary element method (BEM)[8] has emerged as a powerful tool to study problems in engineering practice. The BEM modeling is superior to that of finite elements, especially in zones of stress concentrations. The shear-deformable plate bending boundary element formulation was formulated by Vander Weeën.[9] Rashed[10] extended the formulation of Vander Weeën[9] to model flat plate problems where columns were modeled using the real geometrical cross section. Despite the superiority of the BEM, the moment transfer at slab-column connections in flat plates has never been studied using the BEM.

In this paper, a new method is proposed to compute values of moment transfer parameters. The proposed method is based on using the BEM to model the overall slab. The slab is modeled using the shear deformable plate bending theory according to Reissner.[7] The unbalanced moment is introduced over the studied column's real cross section. The proposed method is implemented into a computer software tool to allow for practical usage. The developed tool is used to study the same problem proposed by Elgabry and Ghali.[6] Several columns are studied, including interior, edge, and corner columns with different rectangularity ratios. The computed values of moment transfer parameters are plotted against values obtained by Elgabry and Ghali[6] and the ACI[1] equations.

RESEARCH SIGNIFICANCE

The proposed method provides a better understanding of the moment transfer mechanism in any slab-column connection. It presents more accurate modeling than the previously published linear stress hypothesis,[1] the Fourier series method,[5] or FEM.[6] It also provides modeling the overall slab and allows the application of the unbalanced moment to be over the considered column cross section. Local straining actions can be easily computed. Therefore, it gives more critical assessment of the accuracy of the ACI 318-08.[1] As an alternative, it can be used via a "software tool" by practicing engineers to compute moment transfer parameters for irregular slabs and columns.

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BOUNDARY ELEMENTS FOR FLAT PLATES

Consider an arbitrary plate of domain Ω boundary Γ, loaded by domain loading of intensity q, as shown in Fig. 1. The indicial notation is used in this section where Greek indexes vary from 1 to 2 (to denote the x and y directions) and Roman indexes vary from 1 to 3 (to denote the x, y, and z directions). The Reissner plate bending theory[7] is used in the present formulation. The plate is supported over a series of columns. It is assumed that columns are connected to the slab over distributed patches (refer to Fig. 1). Three interaction forces are considered: two bending moments in the two directions and one vertical shear. The direct boundary integral equation for such a plate can be written in the following form[10]

$$C_{ij}(\xi)u_j(\xi) + \int_{\Gamma(x)} T_{ij}(\xi, x)u_j(x)d\Gamma(x) = \int_{\Gamma(x)} U_{ij}(\xi, x)t_j(x)d\Gamma(x) \quad (1)$$
$$+ \int_{\Gamma(x)} \left[V_{i,n}(\xi, x) - \frac{v}{(1-v)\lambda^2}U_{ia}(\xi, x)\right]qd\Gamma(x)$$
$$+\sum_c \left\{ \int_{\Omega_c(y)} \left[U_{ik}(\xi, y) - \frac{v}{(1-v)\lambda^2}U_{i\alpha,\alpha}(\xi, y)\delta_{3k}\right]d\Omega_c(y)\right\}$$
$$\times\left[\frac{-u_k(y)S_k(y)}{A(y)} - q\delta_{3k}B(y)\right]$$

where $T_{ij}(\xi, x)$, $U_{ij}(\xi, x)$ are the two-point fundamental solution kernels for tractions and displacements, respectively.[9] The two points ξ and x are the source and field points, respectively. The values $u_j(x)$ and $t_j(x)$ denote the boundary generalized displacements and tractions. The value $C_{ij}(\xi)$ is the jump term and the kernel $V_i(\xi, x)$ is a suitable particular solution to represent domain loading.[9] The symbols ν and λ denote the plate Poisson's ratio and shear factor. The symbol c denotes the number of internal columns that have domain Ω_c. Field Point y denotes the point of the internal column center, and S_k represents the bending and the axial stiffness of the column. The coefficient $B(y)$ can take either zero to represent that the column is ended at the considered floor or one if the column will continue to the top floor. It has to be noted that no summation is considered over the k index in all equations in this section.

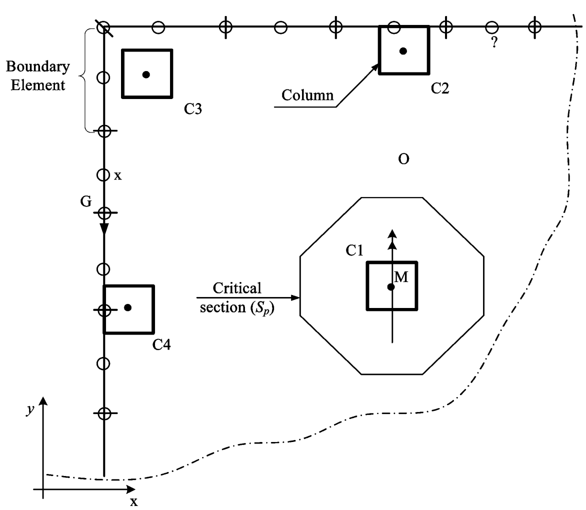


Fig. 1—General flat plate supported on columns.

Equation (1) represents three integral equations. If the slab boundary is discretized into quadratic elements (three nodes per element), Eq. (1) can be rewritten at each boundary node ξ and at each column center to form a system of linear equations. Boundary values $u_j(x)$ and $t_j(x)$, together with generalized displacements at internal column centers, can be obtained from the solution of such a system of equations.

Internal values at any internal point ξ can be computed as a post-processing stage. For example, displacements at internal points can be computed using Eq. (1) with $C_{ij}(\xi) = \delta_{ij}$ (the identity matrix), whereas straining action values (bending and twisting moments $M_{\alpha\beta}$ and shear forces $Q_{3\beta}$) can be computed using other integral equations as follows[10]

$$M_{\alpha\beta}(\xi) = \int_{\Gamma(x)} U_{\alpha\beta k}(\xi, x)t_k(x)d\Gamma(x) - \int_{\Gamma(x)} T_{\alpha\beta k}(\xi, x)u_k(x)d\Gamma(x) \quad (2)$$
$$+ q\int_{\Gamma(x)} W_{\alpha\beta}(\xi, x)d\Gamma(x) + \frac{v}{(1-v)\lambda^2}q\delta_{\alpha\beta}$$
$$+\sum_c \left\{ \int_{\Omega_c(y)} \left[U_{\alpha\beta k}(\xi, y) - \frac{v}{(1-v)\lambda^2}U_{\alpha\beta\theta,\theta}(\xi, y)\delta_{3k}\right]d\Omega_c(y)\right\}$$
$$\times\left[\frac{-u_k(y)S_k(y)}{A(y)} - q\delta_{3k}B(y)\right]$$

and

$$Q_{3\beta}(\xi) = \int_{\Gamma(x)} U_{3\beta k}(\xi, x)t_k(x)d\Gamma(x) - \int_{\Gamma(x)} T_{3\beta k}(\xi, x)u_k(x)d\Gamma(x) \quad (3)$$
$$+ q\int_{\Gamma(x)} W_{3\beta}(\xi, x)d\Gamma(x)$$
$$+\sum_c \left\{ \int_{\Omega_c(y)} \left[U_{3\beta k}(\xi, y) - \frac{v}{(1-v)\lambda^2}U_{3\beta\theta,\theta}(\xi, y)\delta_{3k}\right]d\Omega_c(y)\right\}$$
$$\times\left[\frac{-u_k(y)S_k(y)}{A(y)} - q\delta_{3k}B(y)\right]$$

The relevant new kernel U_{ijk}, T_{ijk}, and $W_{i\beta}$ and their relevant derivatives are given by Rashed.[10] It has to be noted that all kernels at internal points are smooth and could be straight forward computed even over column centers.

PROPOSED METHOD OF CALCULATING γ_v AND γ_f

A proposed method is adapted to compute the moment transfer parameters (γ_v and γ_f) at any slab-column connection. This method is based on the elastic analysis of a slab using the BEM. The BEM is used in the analysis because it deals with the problem as a single continuum domain leading to high accuracy, especially at stress concentration zones. Without losing the generality, the proposed method is used for a circular and rectangular column at different positions (interior, edge, and corner columns).

Consider an arbitrary flat plate, as shown in Fig. 1. Again, without losing the generality, in this section, the proposed procedures are explained on the interior column (C1). Other columns (for example, C2 and C3) can be analyzed using the same strategy. It is required to compute the values of γ_{vy} and γ_{fy} for any desired section S_p, as shown in Fig. 1, due to an unbalanced moment vector in the y-direction (M_x). The following algorithm is proposed:

1. No load is introduced on the slab.
2. A unit moment is introduced on an internal domain patch or cell placed over the Column C1 area ($M_x = 1$). In this case, the rotational stiffness in the direction of the applied moment vector of this column is set to zero (that is, $k_y = 0$). This is done to allow the column to act as an applied load element in this direction.
3. The problem is solved using the boundary element method (Eq. (1)). The value of the Column C1 vertical reaction R is determined. Values of the internal stress resultants (M_{xx}, M_{xy}, M_{yy}, Q_{zx}, and Q_{zy}) for the relevant critical section (S_p) around the column are determined (Eq. (2) and (3)). This analysis case will be referred to as Case 1.
4. To have only a free moment applying over the column, the computed column reaction should vanish (refer to Fig. 2). This is done by introducing the same problem again, but the cell loading in this case is set to be equal to the negative value of the column reaction ($P = -R$). In this case, the vertical stiffness of the column is set to zero ($k_3 = 0$) to allow the column to act as an applied load member in the Z-direction. The column's rotational stiffness in both directions are considered using their actual values. This problem is solved again using the BEM (Eq. (1)), and the values of the stress resultants along the critical section in this case are computed (Case 2). Figure 2 summarizes the previous two steps.
5. The straining action values at the internal points along the relevant critical section (S_p) in the former two cases (Cases 1 and 2) are added together. Hence, an equilibrium analysis of the critical-section free-body diagram is carried out. The total forces generated by shear and its point of application, as well as the total moment generated by bending and twisting moments, are computed. This is done by integration along each critical section segment as follows (refer to Fig. 2)

$$F_{3i} = \int_{S_i} Q_{3i} dS \tag{4}$$

$$\bar{x}_i = \frac{\left(\int_{S_i} Q_{3i} x_i dS\right)}{\left(\int_{S_i} Q_{3i} dS\right)} \tag{5}$$

$$m_{ij} = \int_{S_i} M_{ij} dS \tag{6}$$

where F_{3i} is the resulting vertical force acting on the line segment (S_i) of the critical section (S_p); Q_{3i} is the shear distribution function along the line segments (S_i) of the critical section (S_p); $\bar{x}_i$ is the location of the resulting shearing force acting along the line segment (S_i); m_{ij} is the total bending or twisting moment acting on the line segment (S_i) of the critical section (S_p); and M_{ij} is the bending or twisting moment distribution function along the line segments (S_i) of the critical section (S_p).

It should be noted that each line forming the critical section should be divided into at least two segments. This is to take into account the effect of symmetrical distributions. For example, shear forces on a certain line segment may have symmetrical values leading to zero total shear force but may cause moments. In this work, the integration is carried out numerically using the Gauss quadrature method.[11] The number of Gauss points is left to the engineer or the modeler (the user) to choose. In this paper, 10 Gauss points are used. It has to be noted that the use of Eq. (2) and (3) to compute straining actions produces analytical results; therefore, the used algorithm is regarded as a semi-analytical semi-numerical scheme as numerical integrations are involved.

6. The total moment caused by the shear force, as well as those caused by the twisting and bending moments, are calculated as follows (refer to Fig. 3)

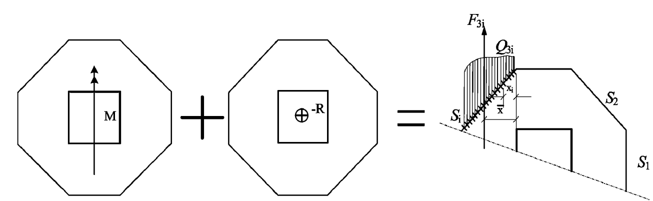
$$M_F = \sum_{S_p} F_{3i} \bar{x}_i \tag{7}$$

$$M_m = \sum_{S_p} m_{ij} \tag{8}$$

where M_F is the total moment on the critical section (S_p) due to the shearing forces; M_m is the total moment on the critical section (S_p) due to the bending and twisting moments; and $\bar{x}_i$ is the horizontal distance from the CG of the column to the point of action of total shearing force (F_{3i}).

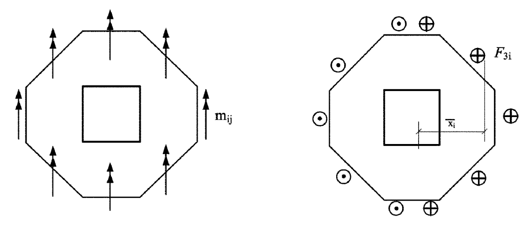
7. Values of γ_v and γ_f can be computed as follows

$$\gamma_{vy} = \frac{\sum(M_F)}{\sum(M_F + M_m)} \tag{9}$$



(a) Case 1: moment only. (b) Case 2: vertical reaction only. (c) Shear distribution along line segment S_i of the critical section (Case 1+Case 2).

Fig. 2—Solution for column with any critical section.



(a): Total twisting and total bending moment along the critical section. (b): Total shear forces over the critical section.

Fig. 3—Straining actions along line segments forming critical section.

$$\gamma_{fy} = \frac{\sum(M_m)}{\sum(M_F + M_m)} \tag{10}$$

It has to be noted that

$$\gamma_{vy} + \gamma_{fy} = 1 \tag{11}$$

where γ_{vy} is the shear transfer parameter about the y-axis; and γ_{fy} is the moment transfer parameter about the y-axis.

The same procedures are repeated for any other critical section. It has to be noted that in some cases (symmetrical cases), the value of the column vertical reaction vanishes. In such a case, there is no need to consider Case 2.

A simple software tool is developed based on the previously mentioned steps to calculate the values of the unbalanced moment parameters (γ_v and γ_f) for any column shape and any position of the critical section.

APPLICATION

The flat slab rested on columns is considered, as shown in Fig. 4. The span of the slab L is 32.81 ft (10 m); the slab thickness t is 0.78 ft (0.24 m); d is the effective depth of value 0.69 ft (0.21 m); Young's modulus E is 4.3 H 10^8 lb/ft^2 (2.1 H 10^6 t/m^2); and Poisson's ratio ν is 0.16. The supporting columns only exist below the slab and of height 13.12 ft (4 m). The same problem was previously considered by Elgabry and Ghali[6] and is reconsidered herein for the sake of comparison. Analyses are done for square, rectangular, and circular columns having different dimensions. Different locations of columns (interior, edge, and corner columns) with different critical section locations (at distance $d/2$ and $2d$ from the column face) are considered. In the analysis, three boundary elements are used to model each side of the problem. It has to be noted that the analysis is carried out in the elastic range. According to Elgabry and Ghali,[6] a reduction in the value of γ_v (the portion of the moment transferred by shear) by 15% is introduced to take into account the effect of cracks that are present in the plastic range. The same assumption is reconsidered in this work.

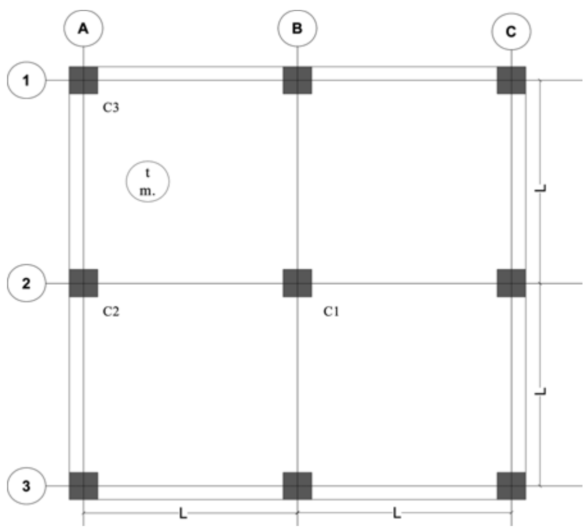


Fig. 4—The analyzed problem.

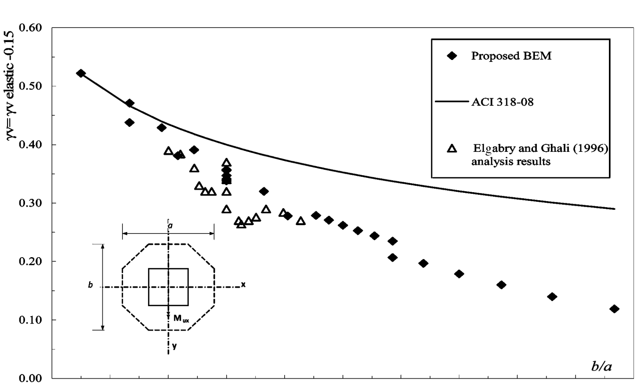


Fig. 5—Computed values of γ_v for interior columns.

Interior column

For the interior column, Eq. (12) is used to compute the values of γ_{vy} and γ_{fy} according to the ACI 318-08[1] (Eq. (11-39) and (13-1) in ACI 318-08[1]) and Elgabry and Ghali[6]

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{(a/b)}} \tag{12}$$

where γ_{vy} is the portion of the unbalanced moment transferring by shear; α is the projected length of the critical section on the x-axis; and β is the projected length of the critical section on the y-axis.

Figure 5 demonstrates the variation in the value of γ_v against the ratio for rectangular interior columns. Values obtained from ACI 318-08,[1] as well as the values obtained from Elgabry and Ghali[6] together with their proposed envelope equation, are plotted on the same graph (Fig. 5). It can be seen that values of γ_v calculated using the ACI 318-08[1] equation give higher values when compared to the results obtained from the proposed analysis. It is clear that by increasing the ratio b/a, the value of the unbalanced moment transferred by shear decreases. This is because increasing the value of b/a leads to the increase of the value of the column stiffness k_y. This increase leads to increase in the column resistance to the applied moment M_x. Therefore, the value of γ_f is increased; consequently, the value of γ_v is decreased. Some differences exist between the proposed BEM analysis and the results of Elgabry and Ghali.[6] It is considered that the proposed BEM model is more accurate for the following reasons:

1. In the BEM model, the unbalanced moment is introduced as a free moment acting over the analyzed column, whereas in work by Elgabry and Ghali,[6] the moment was modeled by prescribing values for the rotations and deflection at nodes forming the column boundary. It is well known that in the FEM analysis it is not preferred to introduce concentrated rotations over nodes as this leads to high local stress concentrations and, consequently, results in loss of accuracy.

2. In the BEM, the lines of the critical sections under study are correctly modeled (that is, the exact shape is considered). The solution along the desired critical section is continuous, as it was obtained from a solution of a continuous integral equation,[10] whereas in Elgabry and Ghali,[6] the solution for any critical section is computed at the nodes. In their analysis,[6] only a few nodes were taken per line (four total), which is small for such a case of shear stress concentrations. This leads to approximate, continuous moment and shear function along the critical section perimeter. These functions are evaluated using coarse step-wise constants, that is, a series of constant functions over successive intervals.[12]

Edge column

For an edge column, the analysis is done in two orthogonal directions. The first case (Case I) is when the moment vector is parallel to the free edge, whereas the other analysis direction (Case II) is when the moment vector is perpendicular to the edge. The same procedure used for interior columns is repeated herein, except that the values of γ_v and γ_f are computed separately for the two cases (Cases I and II).

In Case I, ACI 318-08[1] does not differentiate between interior and edge columns. The same equation used to compute γ_v for interior columns is used in this case (Eq. (12)). This problem was studied by Elgabry and Ghali[6] based on the FEM numerical results. They proposed an envelope equation for the unbalanced moment transfer as follows

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{(a/b) - 0.2}} \quad (a/b \geq 0.2) \qquad (13)$$

$$\gamma_{vy} = 0.4 \quad (a/b < 0.2) \qquad (14)$$

Figure 6 demonstrates the variation of γ_{vy} against the ratio b/a for the edge column. The values of the unbalanced moment transfer calculated by Elgabry and Ghali,[6] as well as ACI 318-08[1] are larger than those obtained from the proposed BEM model. This might be due to the same previously mentioned reasons as in the case of interior column.

In Case II, both the ACI 318-08[1] and Elgabry and Ghali[6] proposed the same value of γ_{vx}, which is similar to that of the interior column (refer to Eq. (12)). The present analysis for this case is carried out using the proposed BEM model and the obtained results are plotted together with the ACI 318-08[1] and Elgabry and Ghali[6] in Fig. 7.

It can be seen from Fig. 7 that the proposed equation by ACI 318-08[1] and Elgabry and Ghali[6] overestimates the value of the moment transfer by shear. The difference in the results between the proposed BEM model and ACI 318-08[1] or Elgabry and Ghali[6] methods are due to the considered shape of the critical section, as Eq. (12) considers only three sides for the critical section.

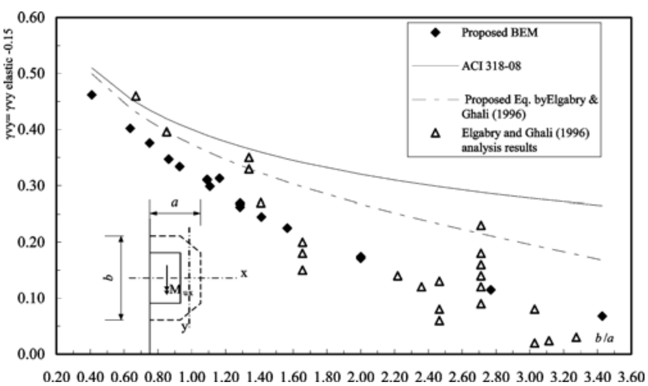


Fig. 6—Computed values of γ_v for edge columns when moment vector is parallel to slab edge.

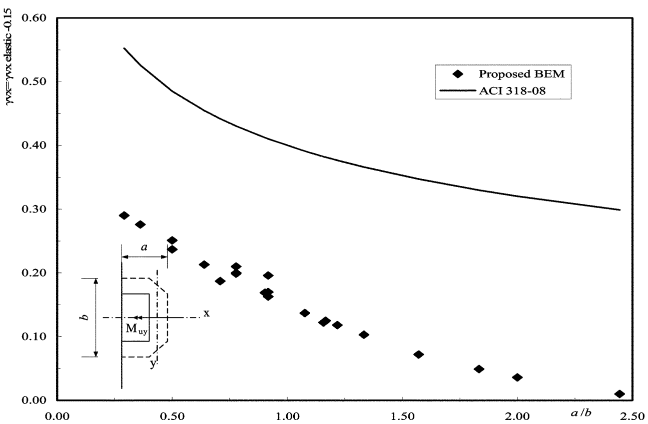


Fig. 7—Computed values of γ_v for edge columns when moment vector is perpendicular to slab edge.

Corner column

A corner column is similar to an edge column. It needs to be studied in the following two principal directions:

1. When the moment vector passes the CG of the column and along the maximum principal axes of the critical section (Case I).
2. When the moment vector is in a direction perpendicular to the one in the previous case (Case II).

The previously mentioned directions (principal directions) are the same directions as those studied by Elgabry and Ghali.[6] This is to allow the study of the moment in each direction separately (without the presence of moment transfer in both directions).

The obtained results of γ_v, based on the proposed BEM model, is plotted against the equation given by Elgabry and Ghali[6] and ACI 318-08[1] for each direction.

In Case I, the equation of ACI 318-08[1] used in the calculation of the value of γ_v is the same as that for interior

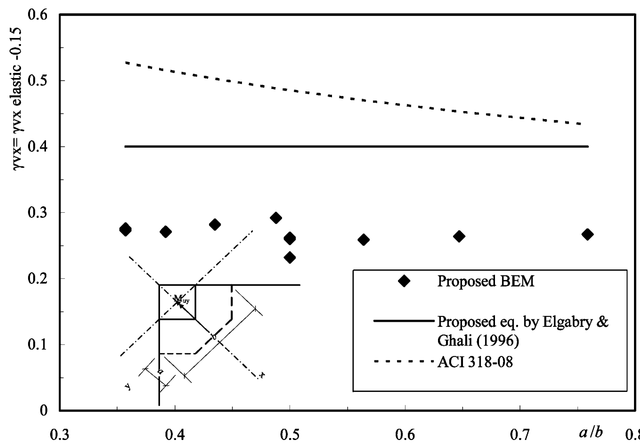


Fig. 8—Computed values of γ_v for corner columns when moment vector is passing CG of principal axes of critical section.

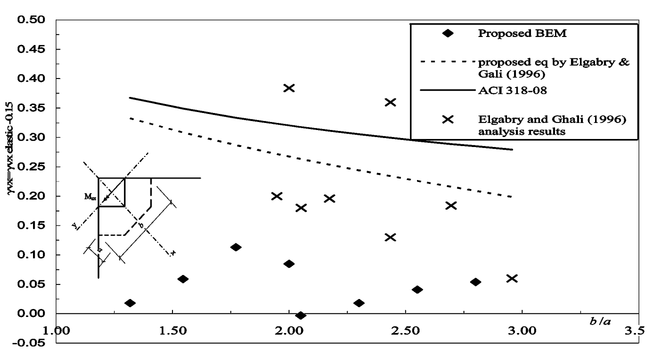


Fig. 9—Computed values of γ_v for corner columns when moment vector is perpendicular to CG of critical section.

columns (Eq. (12)). In the analyses of Elgabry and Ghali,[6] however, the proposed value of γ_v is different. They proposed an envelope equation for their analysis results in such a case as follows

$$\gamma_{vx} = 0.4 \quad (15)$$

Figure 8 demonstrates the variation of γ_{vx} against the ratio a/b. Results of the proposed BEM model indicate that by increasing the ratio a/b, the values of γ_{vx} are almost constant. This agrees with the proposed equation given by Elgabry and Ghali.[6] The proposed BEM model analysis, however, gives a constant value at approximately 0.29 (as an upper envelope), unlike the value of 0.4 proposed by Elgabry and Ghali.[6]

In Case II, ACI 318-08[1] also uses the same equation for the interior column (Eq. (12)). Elgabry and Ghali[6] proposed the use of Eq. (13) and (14) for the edge column, in which the moment vector is parallel to the free edge to be used in this case. The values of the envelope of Elgabry and Ghali[6] and the ACI 318-08,[1] together with the proposed BEM model results, are plotted together in Fig. 9. It can be seen that in this case, according to the present BEM results, the moment transfer is mainly governed by interior bending and twisting moments along the critical section lines rather than shear.

CONCLUSIONS

In this paper, a new method for computing γ_v and γ_f was presented. The method was based on modeling the slab-column connection using the BEM via the Reissner plate theory. The column real cross-section was considered. Semi-analytical-semi-numerical procedures were used to compute such values. An application problem was analyzed using the present method. The following conclusions might be drawn:

1. The present method is easy to be used by practicing engineers via a developed simple software tool.

2. The ACI 318[1] generally overestimates the values of γ_v, especially when the rectangularity ratio of the column increases. For example, for interior columns with b/a = 1, 2, and 2.5, the ACI value of γ_v is 0.40, 0.32, and 0.30, whereas the proposed values are 0.36, 0.18, and 0.13, respectively.

The present method can be easily used to compute γ_v and γ_f for irregular column shapes.

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