Where are the source points?

A brief presentation on the utilization of genetical algorithms and ANNs to optimize the source points' location in the method of fundamental solutions.

Prepared by,
Eng. Ahmed Mahmoud Mostafa Hasan

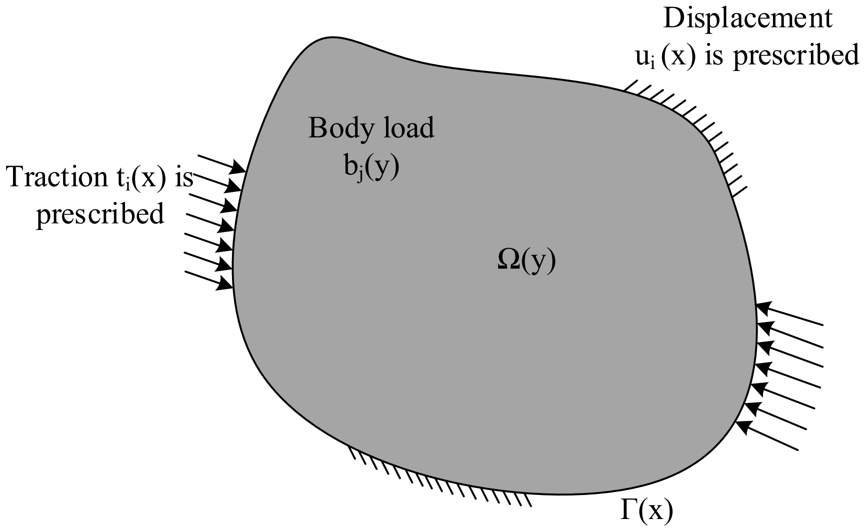
Under supervision from

Prof Dr. Y. F. Rashed

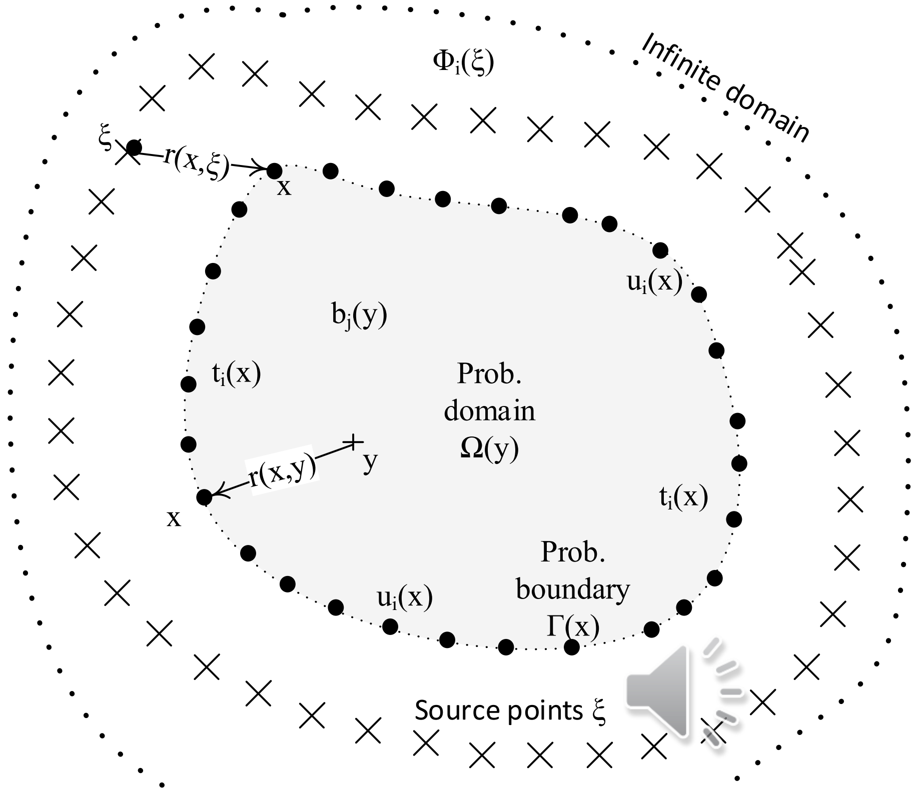
Table of content

Introduction (MFS, ECond, GA, ANN)

- The Method of fundamental solutions MFS



A) Real Problem



B) Virtual Problem

$$u_i(\mathrm{x}) = \sum_{k=1}^{N} U_{ij}^{*}(\mathrm{x}, \xi_k)\phi_j(\xi_k) + \int_{\Omega(\mathrm{y})} U_{ij}^{*}(\mathrm{x},\mathrm{y}) b_j(\mathrm{y})\, d\Omega(\mathrm{y}) \qquad (1)$$

$$t_i(\mathrm{x}) = \sum_{k=1}^{N} T_{ij}^{*}(\mathrm{x}, \xi_k)\phi_j(\xi_k) + \int_{\Omega(\mathrm{y})} T_{ij}^{*}(\mathrm{x},\mathrm{y}) b_j(\mathrm{y})\, d\Omega(\mathrm{y}) \qquad (2)$$

Introduction (MFS, ECond, GA, ANN)

- The Method of fundamental solutions MFS

$$\{u\}_{2N\times1} = [U^*]_{2N\times2N}\{\phi\}_{2N\times1} + \{u_p\}_{2N\times1}$$

$$\{t\}_{2N\times1} = [T^*]_{2N\times2N}\{\phi\}_{2N\times1} + \{t_p\}_{2N\times1}$$

$$\{t\}_{2N\times1} = [T^*]_{2N\times2N}[U^*]^{-1}{}_{2N\times2N}\left(\{u\}_{2N\times1} - \{u_p\}_{2N\times1}\right) + \{t_p\}_{2N\times1}$$

$$[L]_{2N\times2N}\{t\}_{2N\times1} = [L]_{2N\times2N}[T^*]_{2N\times2N}[U^*]^{-1}{}_{2N\times2N}\left(\{u\}_{2N\times1} - \{u_p\}_{2N\times1}\right) + [L]_{2N\times2N}\{t_p\}_{2N\times1}$$

$$\{F\}_{2N\times1} = [K]_{2N\times2N}\left(\{u\}_{2N\times1} - \{u_p\}_{2N\times1}\right) + \{F_p\}_{2N\times1}$$

$$[\boldsymbol{A}]_{2N\times2N}x_{2N\times1} = \{b\}_{2N\times1} \qquad (3)$$

Introduction (MFS, ECond, GA, ANN)

- The effective condition number ECond.

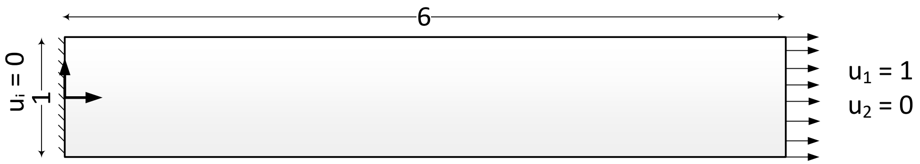
For a numerical system

$$Ax = b$$

the ECond is calculated as

$$ECond = \frac{\|b\|}{\sigma_n \|x\|}$$

where σ_n is the smallest Singular Value Decomposition (SVD) value for A.



Introduction (MFS, ECond, GA, ANN)

- Genetical algorithm (GA) are computational optimization techniques inspired by natural selection and genetics. They work by evolving a population of candidate solutions through processes such as selection, crossover, and mutation, gradually improving solution quality over successive generations.

- Artificail Neural Network (ANN) are computational models inspired by the structure and functioning of the human brain. They consist of interconnected layers of artificial neurons that process and transmit information, enabling the network to learn patterns from data. ANNs are widely used in machine learning for tasks such as pattern recognition, prediction, and decision-making.

Framework for Preprocessor and Postprocessor

<u>Preprocessor</u>

From Equations 1 and 2 formulate and solve equation 3
Calculate all unknown boundary values at boundary points

<u>Postprocessor</u>

Using the known boundary values at boundary points from the **preprocessor** solve equations 1 and 2 for the field points instead of boundary points

Problem with Source Location in MFS

- The source location effect in our presented scheme appears in 2 locations:

1. The $[U^*]$ and $[T^*]$ matrices are calculated based on the location of each source point to the boundary point.

Location change for these sources causes changes in the equation:

$$\boldsymbol{A}\, x = b$$

And eventually effect the condition number for the whole problem.

This step calculates the unknown boundary values for all boundary nodes.

Problem with Source Location in MFS

2. For the domain result calculations, the initial equations are utilized since all unknown boundary values are calculated from the previous step:

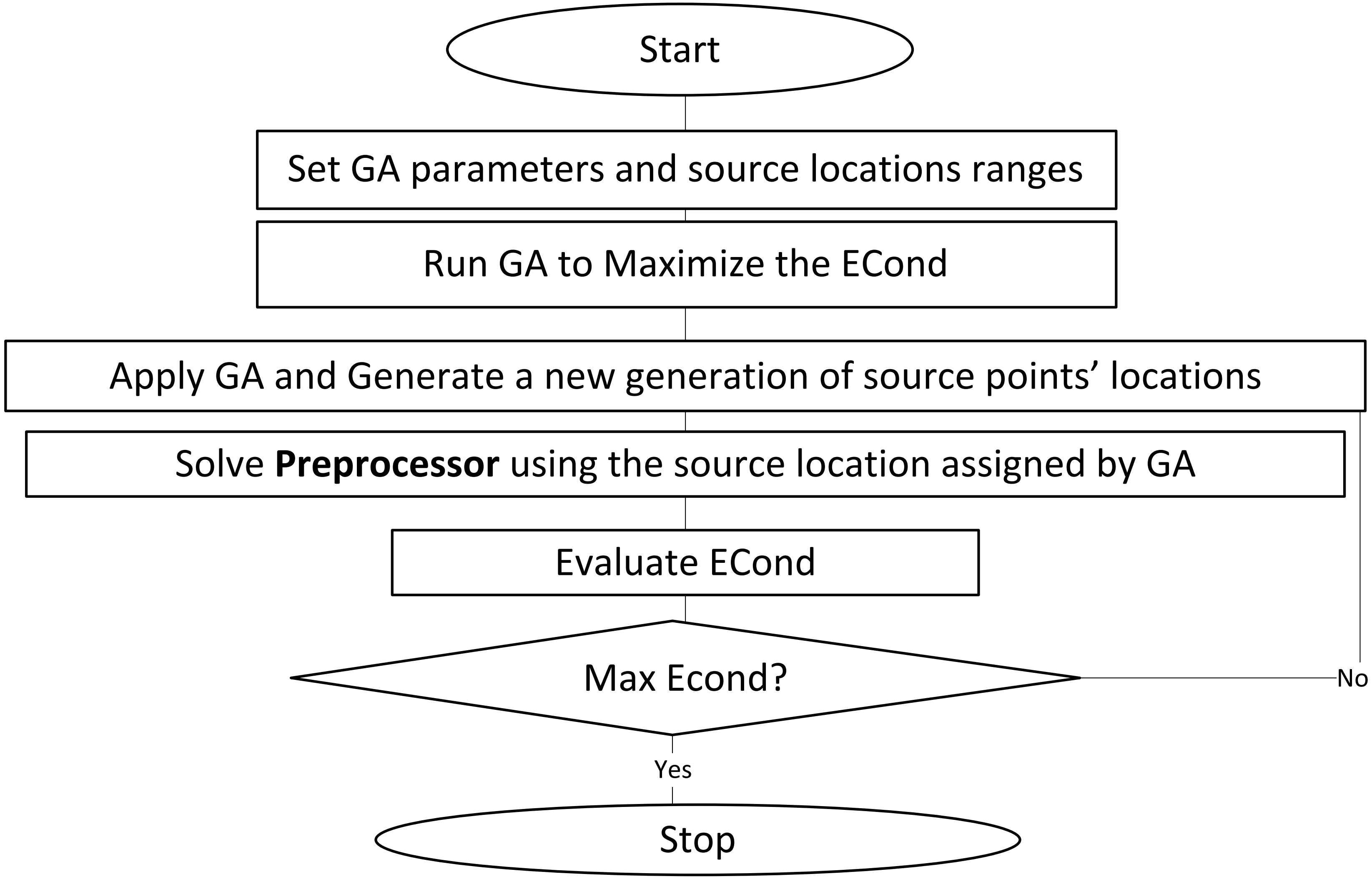
$$\{u\}_{2N\times 1} = [U^*]_{2N\times 2N}\{\phi\}_{2N\times 1} + \{u_p\}_{2N\times 1}$$

$$\{t\}_{2N\times 1} = [T^*]_{2N\times 2N}\{\phi\}_{2N\times 1} + \{t_p\}_{2N\times 1}$$

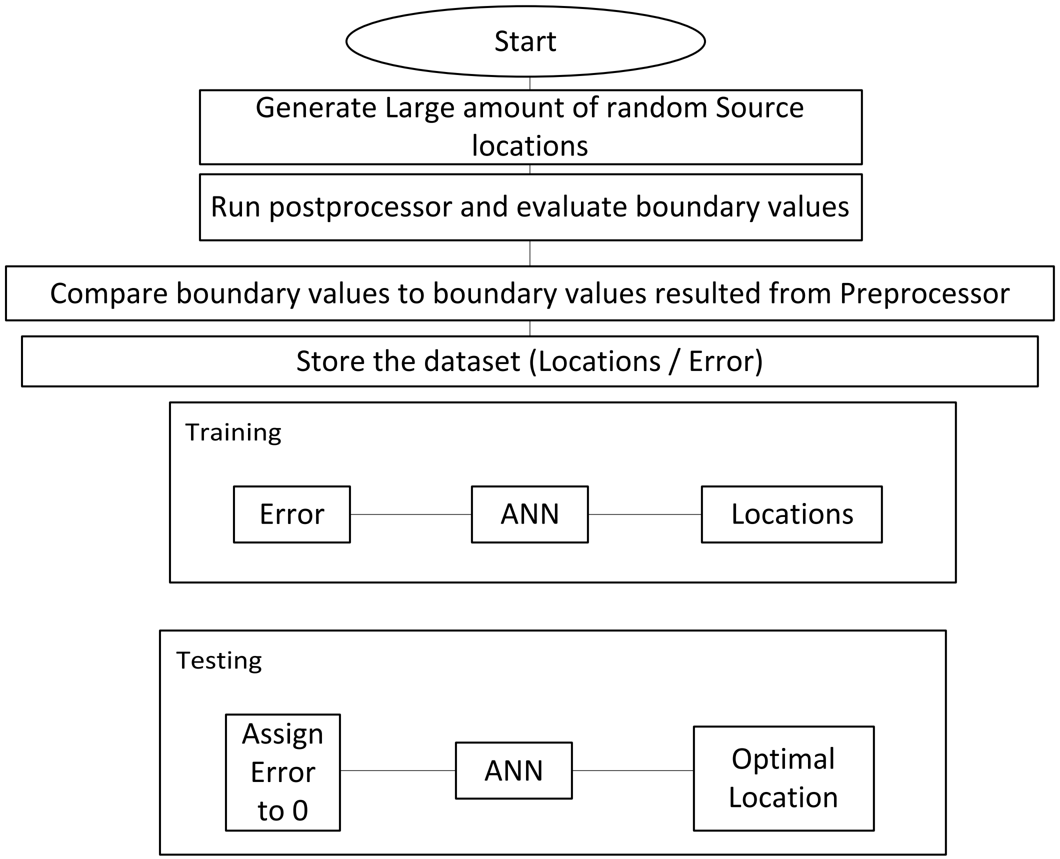
First we solve these problems for $\{\phi\}$ then solve the same equation for any domain location.

In this part the location of the source points effects the intensity values $\{\phi\}$

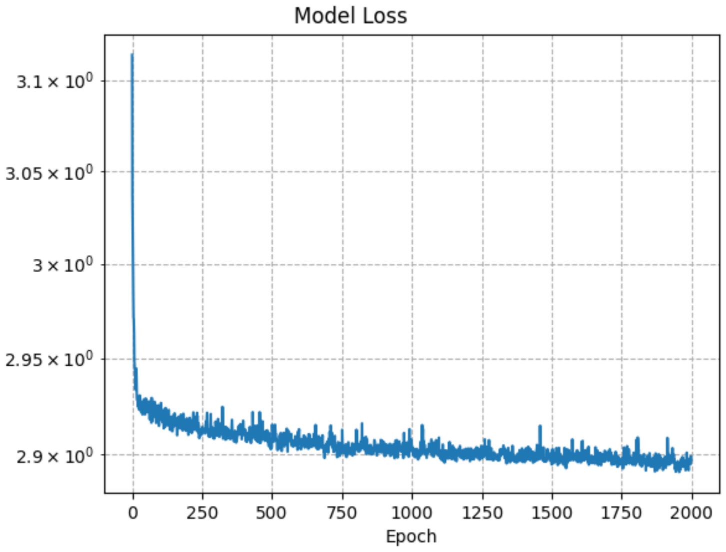
(**Preprocessor**) GA-ECond number Optimization



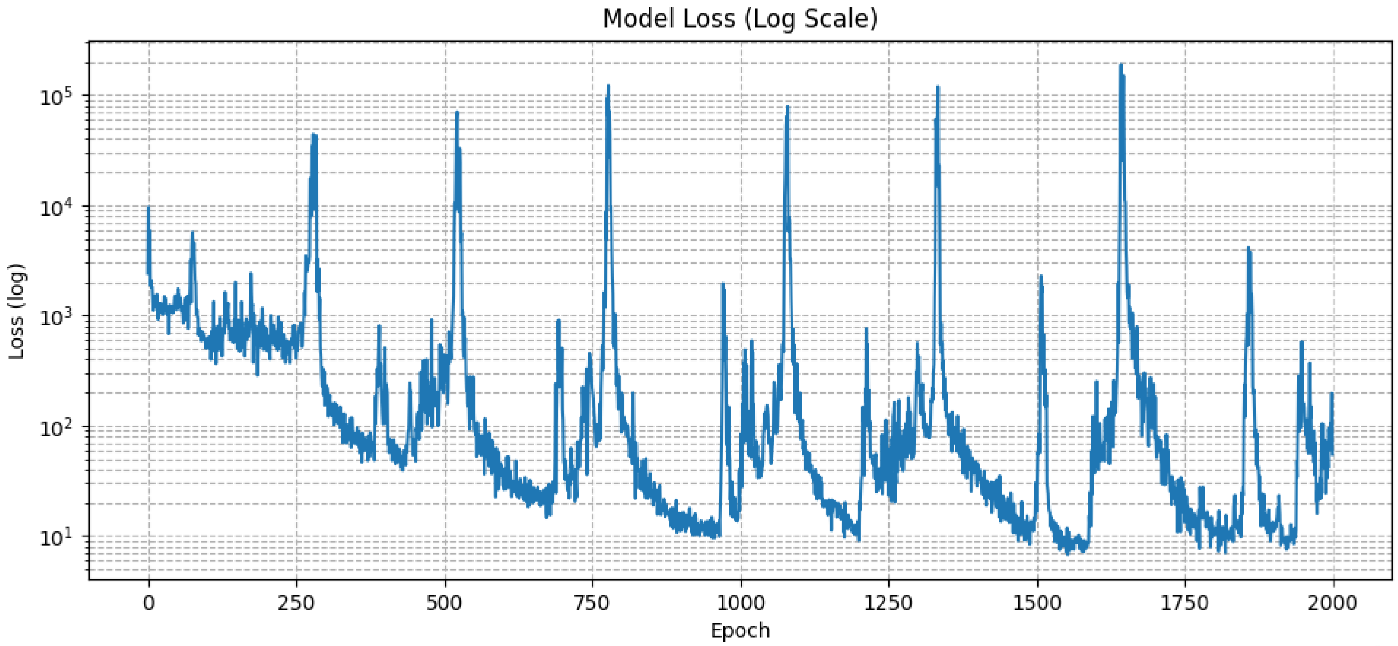
(**Postprocessor**) ANN Supervised



(**Postprocessor**) ANN Supervised

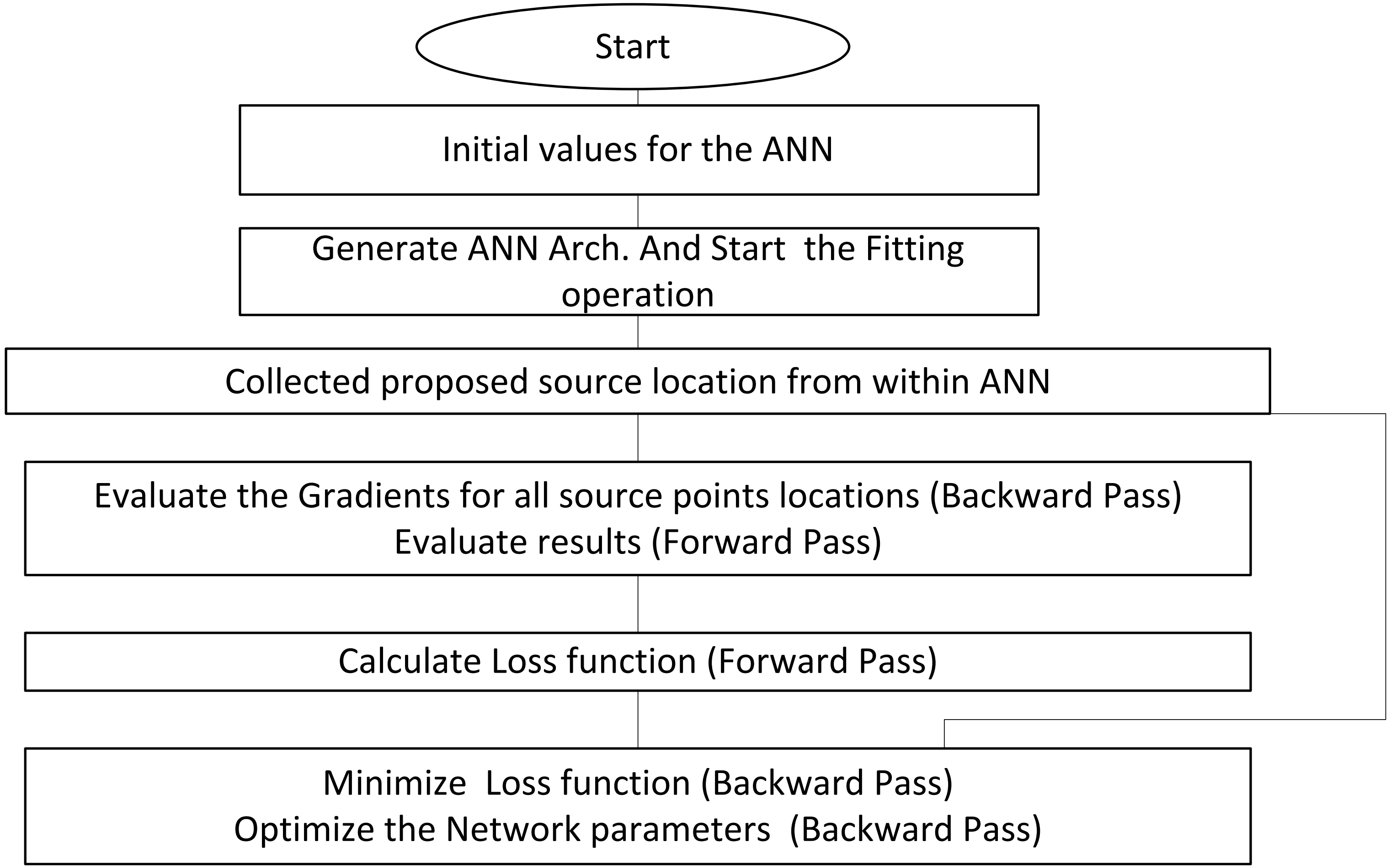


With Normalization

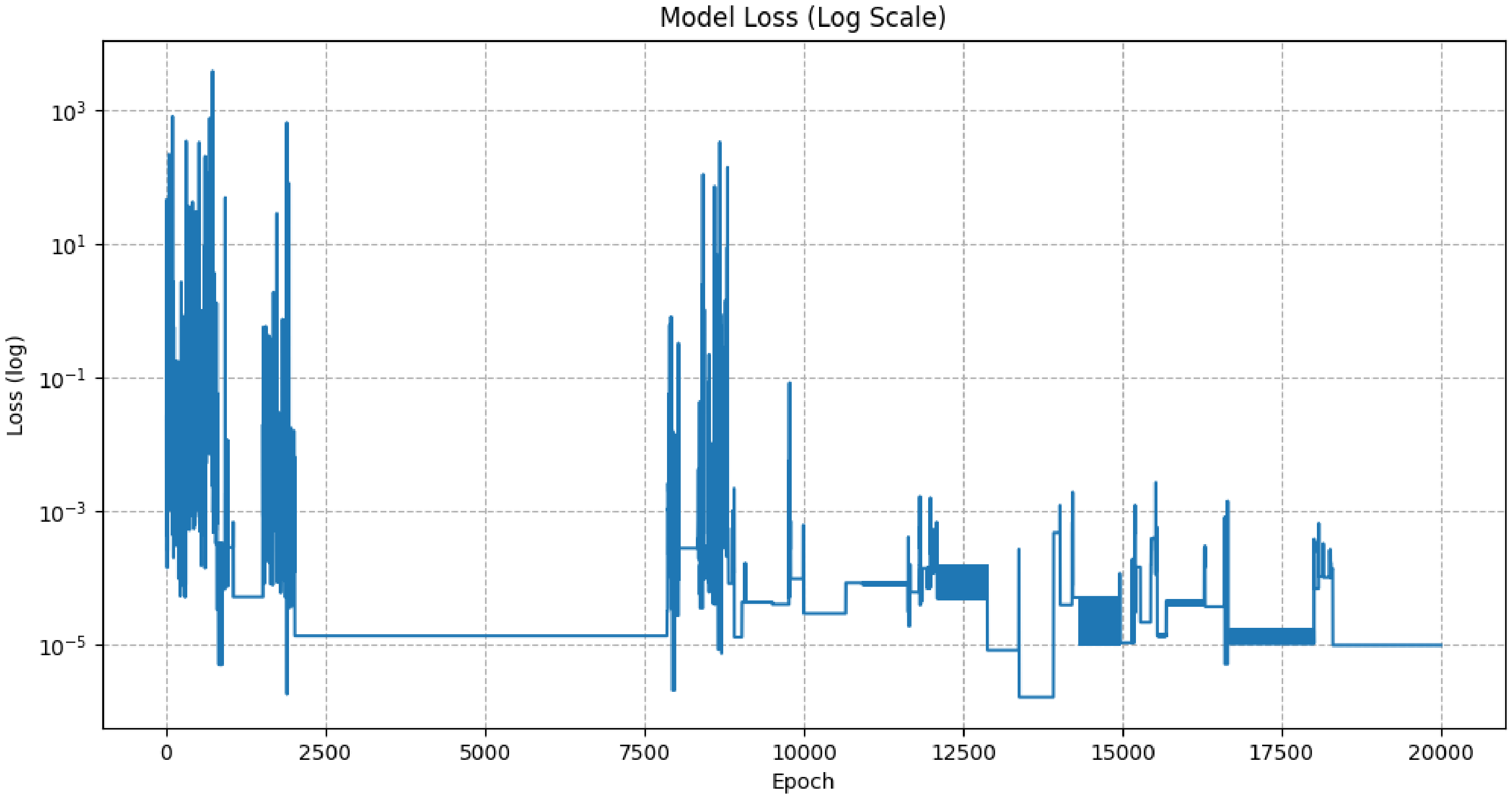


No Normalization

(**Postprocessor**) ANN Unsupervised



(**Postprocessor**) ANN Unsupervised



Conclusion and Challenges

- The proposed framework for optimizing source locations has demonstrated **functionality** and promising operational results.
- However, to fully realize its potential, further refinement of the genetic algorithm and artificial neural network parameters is necessary.
- **Fine-tuning** aspects such as population size, mutation rates, network architecture, and learning rates will help enhance **solution quality**.