بسم الله الرحمن الرحيم

Local and non-local damage modelling using Eshelby inclusion theory

By:

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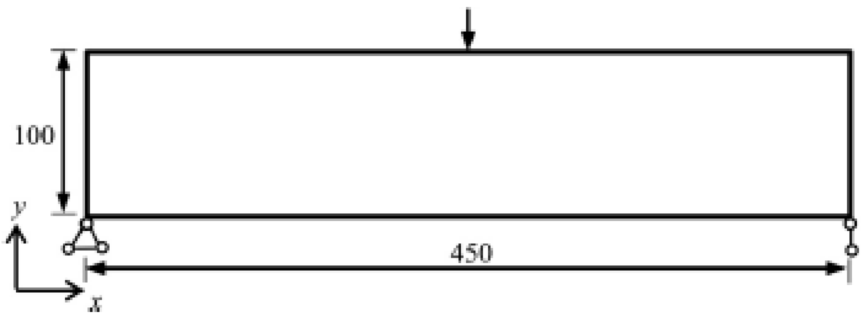
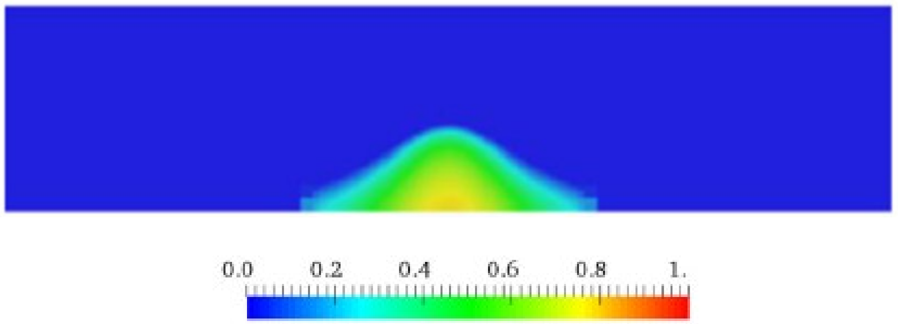
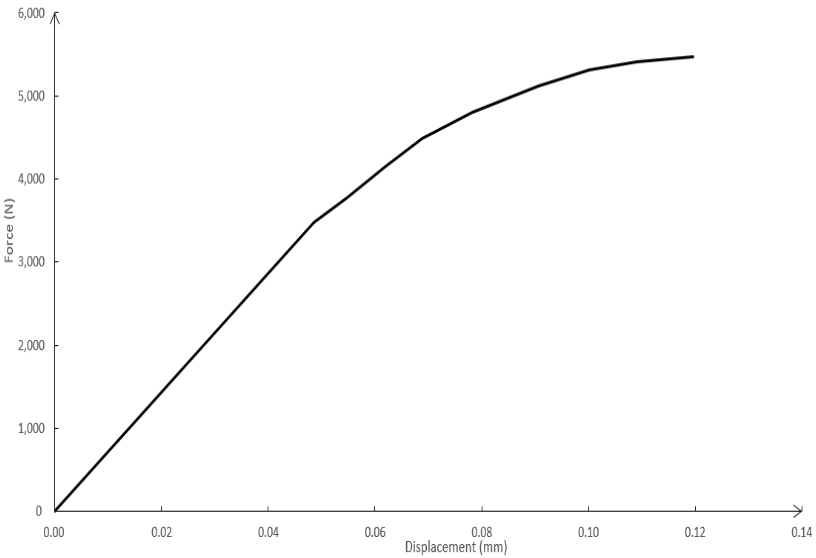
- Introduce a new damage modeling using Eshelby theory of equivalent inclusions coupled with direct boundary integral equation for 2D elasticity problems.
- Boundary only discretization is needed, despite of the damage inside the domain.
- A finite-element like stiffness matrix is formed which is obtained directly in a condensed form on the boundary.

Continuum damage mechanics

It is the study of stiffness degradation of the medium under certain load from continuum mechanics point of view







The damage is modeled using damage variable

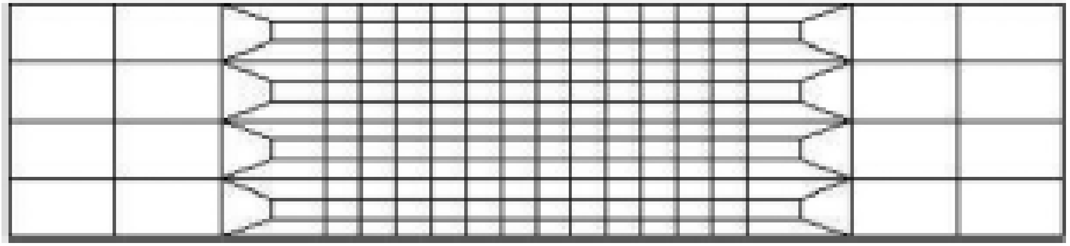
$$C_{ijkl}^{D} = (1 - D)C_{ijkl}$$

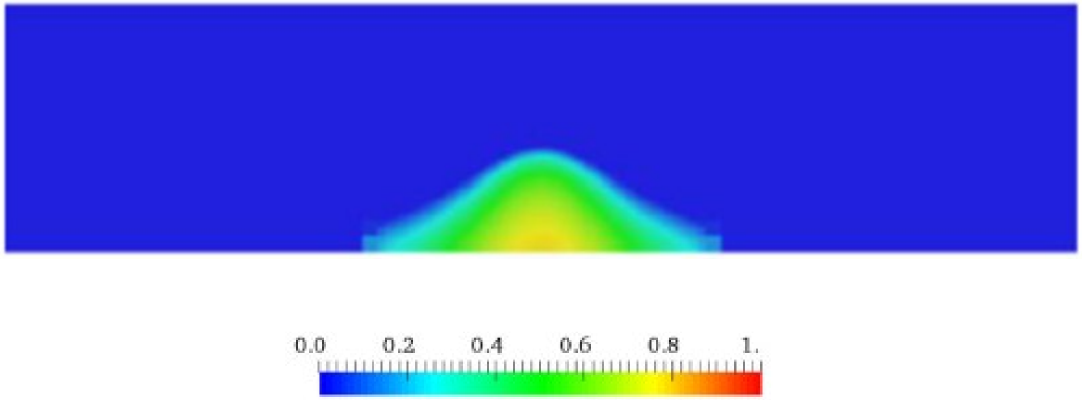
Assumptions:

1. **Damage is isotropic and elastic.**
2. **The material is quasi brittle (damage is due to tensile strain only).**
3. **Poisson ratio is assumed constant**

Damage Modeling

In **Finite element method** the problem is straight forward as the domain is discretized





In the **Boundary element method** the problem is not direct as the boundary only is discretized.

Solution:

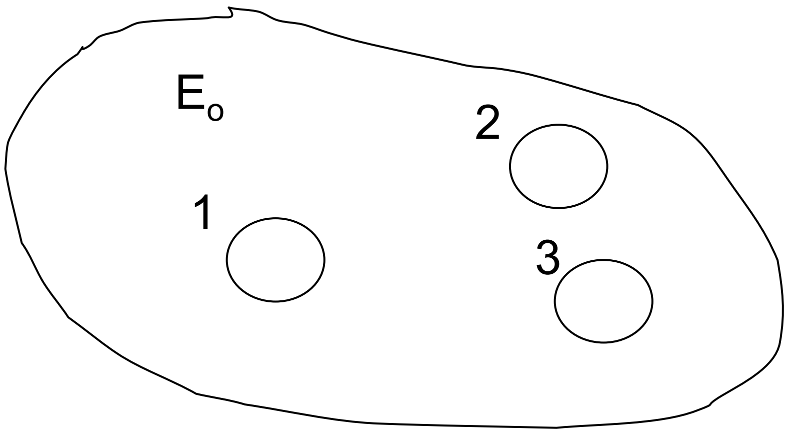
1-Coupling with **Finite element method**.

2- Using Subregions

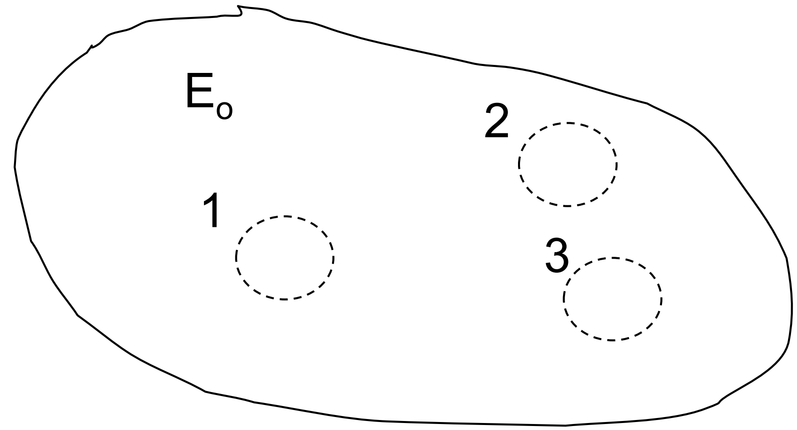
3- Using Cells with applied initial stress or strain

Eshelby equivalent inclusion theory

- **Eshelby solves the problem of inhomogeneity by solving the problem as a homogeneous one with a prescribed strain (eigenstrain) applied at the inhomogeneity locations.**



Problem with inhomogeneities



Homogeneous problem with equivalent inclusions

$$\{\varepsilon_{im}\}^C = \left[S_{imjk}\right]\{\varepsilon_{jk}^o\}$$

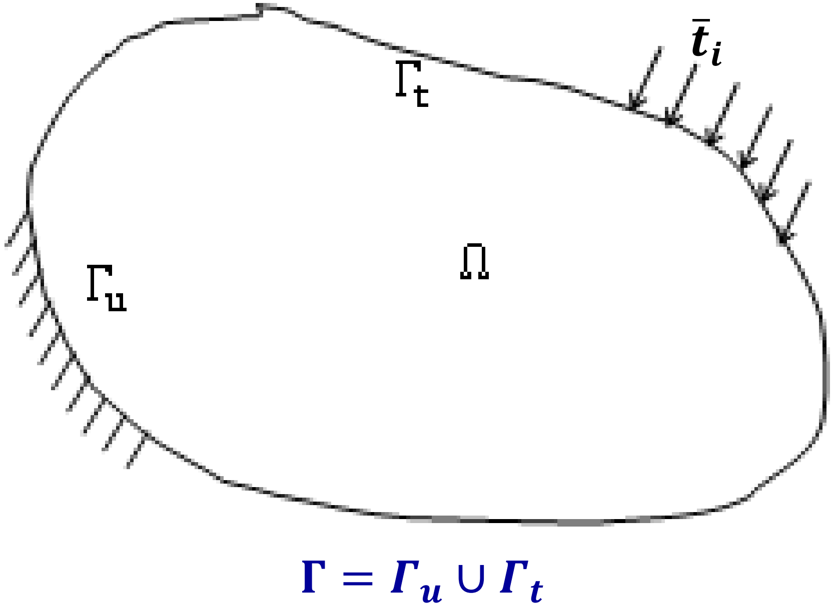
Elasticity equations

$\sigma_{ij,j} = 0$ Equilibrium equation

$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$ Hook’s Law

With the presence of eigenstrain:

$\sigma_{ij,j} = \sigma^{o}_{ij,j}$ Equilibrium equation



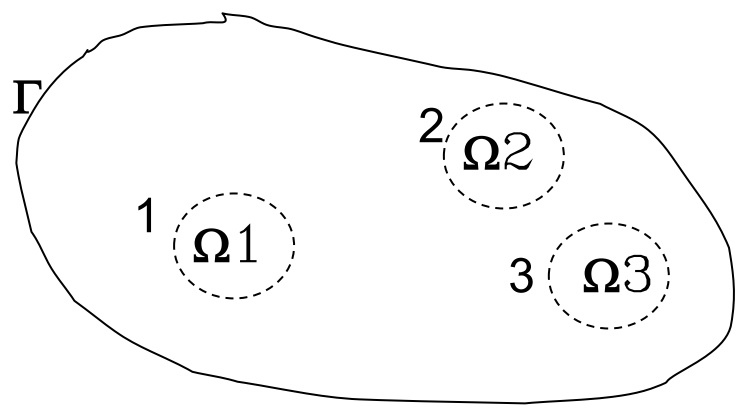
Application of Eshelby theory to direct BEM

- **Displacement boundary integral equation**

$$c_{ij}(\xi)u_j(\xi) = \int_{\Gamma} U^*_{ij}(\xi,x)t_j(x)d\Gamma(x) - \int_{\Gamma} T^*_{ij}(\xi,x)u_j(x)d\Gamma(x) + \sum_{I=1}^{I=NOI} \varepsilon^{o}_{jk}(x_I) \int_{\Omega_I} \sigma^*_{ijk}(\xi,x_I)d\Omega_I(x_I)$$

- **Strain boundary integral equation**

$$\varepsilon_{im}(\xi) = \int_{\Gamma} U^*_{ijm}(\xi,x)t_j(x)d\Gamma(x) - \int_{\Gamma} T^*_{ijm}(\xi,x)u_j(x)d\Gamma(x)$$



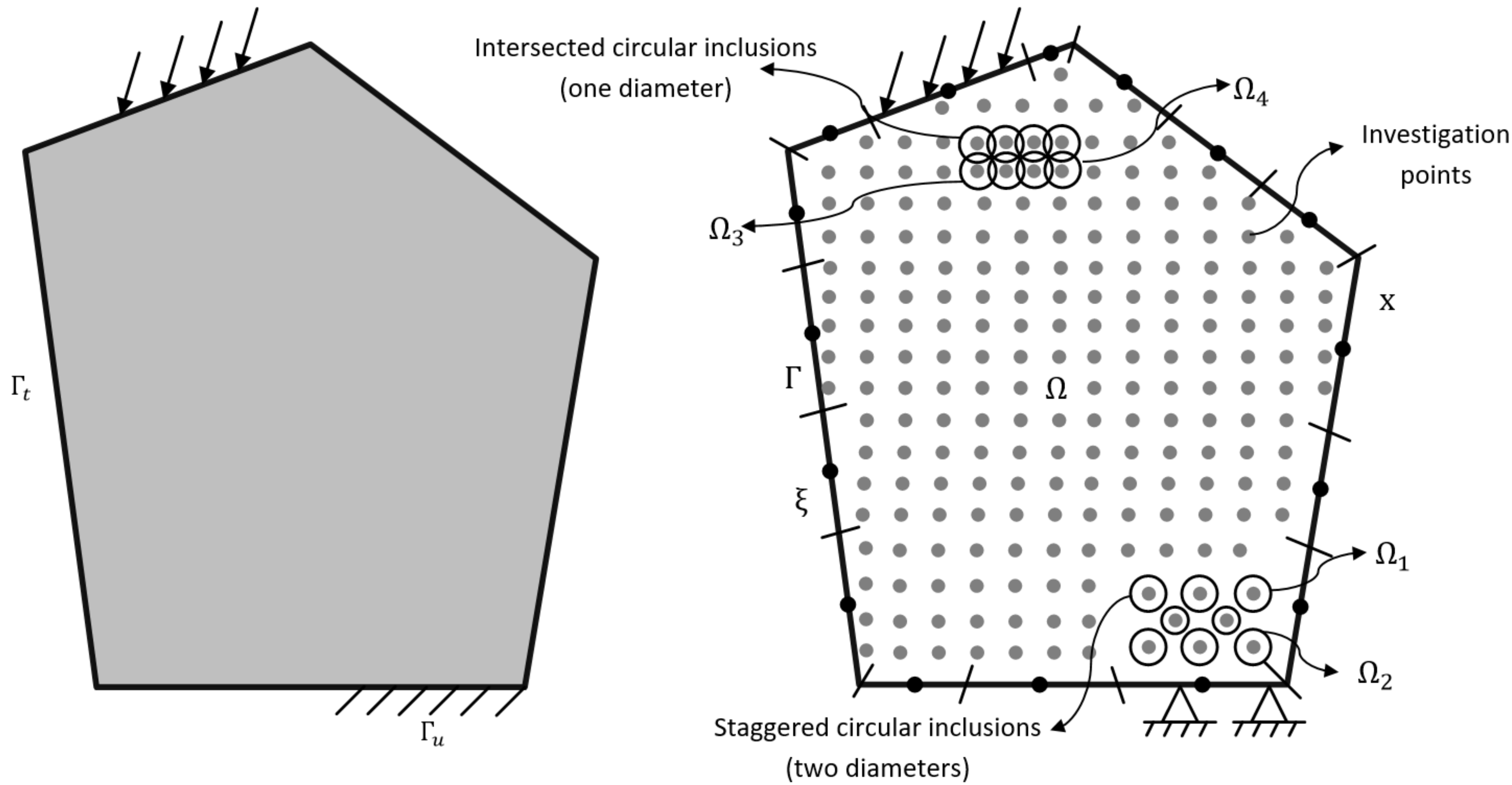
- **Relation between the applied strain and the eigenstrain**

$$\{\varepsilon_{im}\}^{applied} = [ek_{imjk}]\{\varepsilon^{o}_{jk}\}$$

- Rearrange the above three equations to be:

$$\{F\} = [K]\{u\}$$

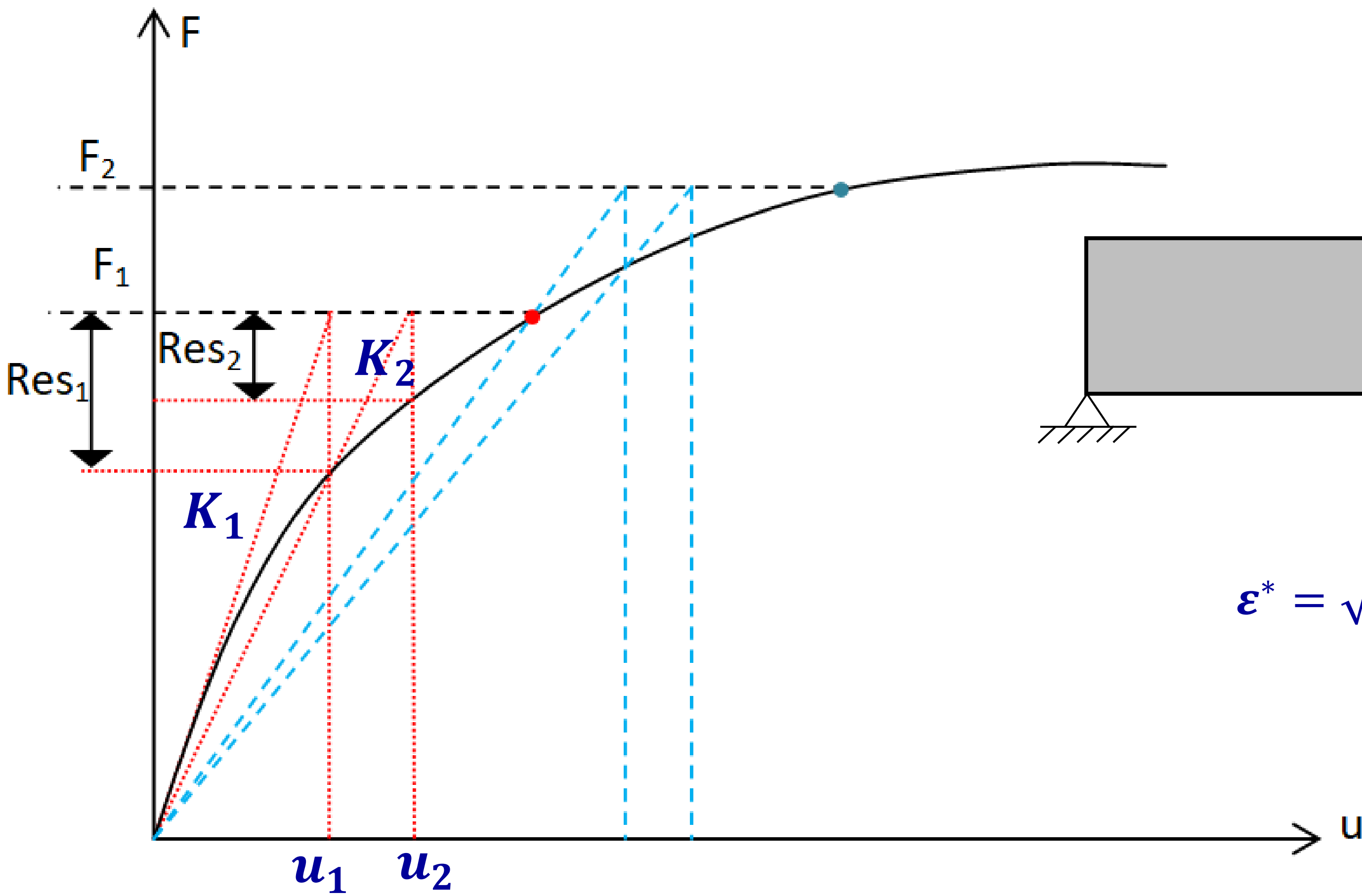
- $[K]$ is the stiffness matrix of the damaged domain obtained directly on the boundary (in a condensed form).
- As the elastic properties of the problem changes at each load step so the problem is nonlinear.
- At each load step the system of equations is solved using a nonlinear solution technique.

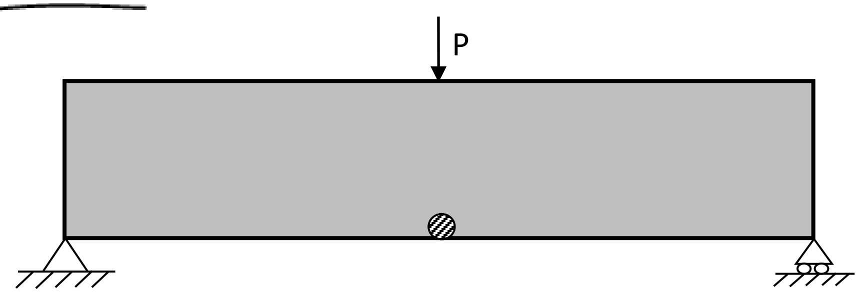


a) Actual problem

b) Discretized problem

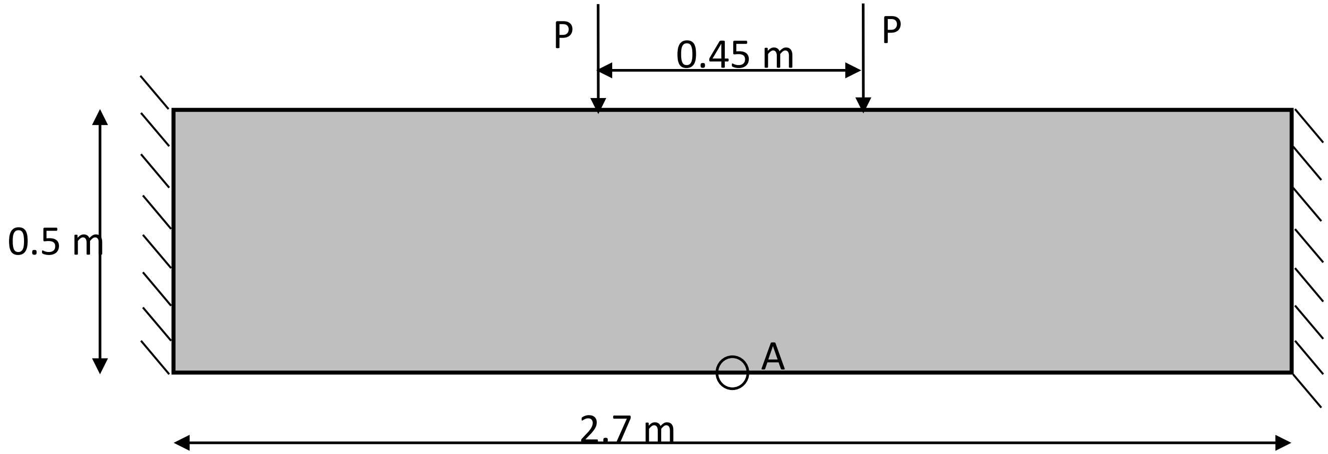
Solution Algorithm:





$$\varepsilon^* = \sqrt{\langle \varepsilon_1 \rangle^2 + \langle \varepsilon_2 \rangle^2}$$

Fixed-Fixed Beam:

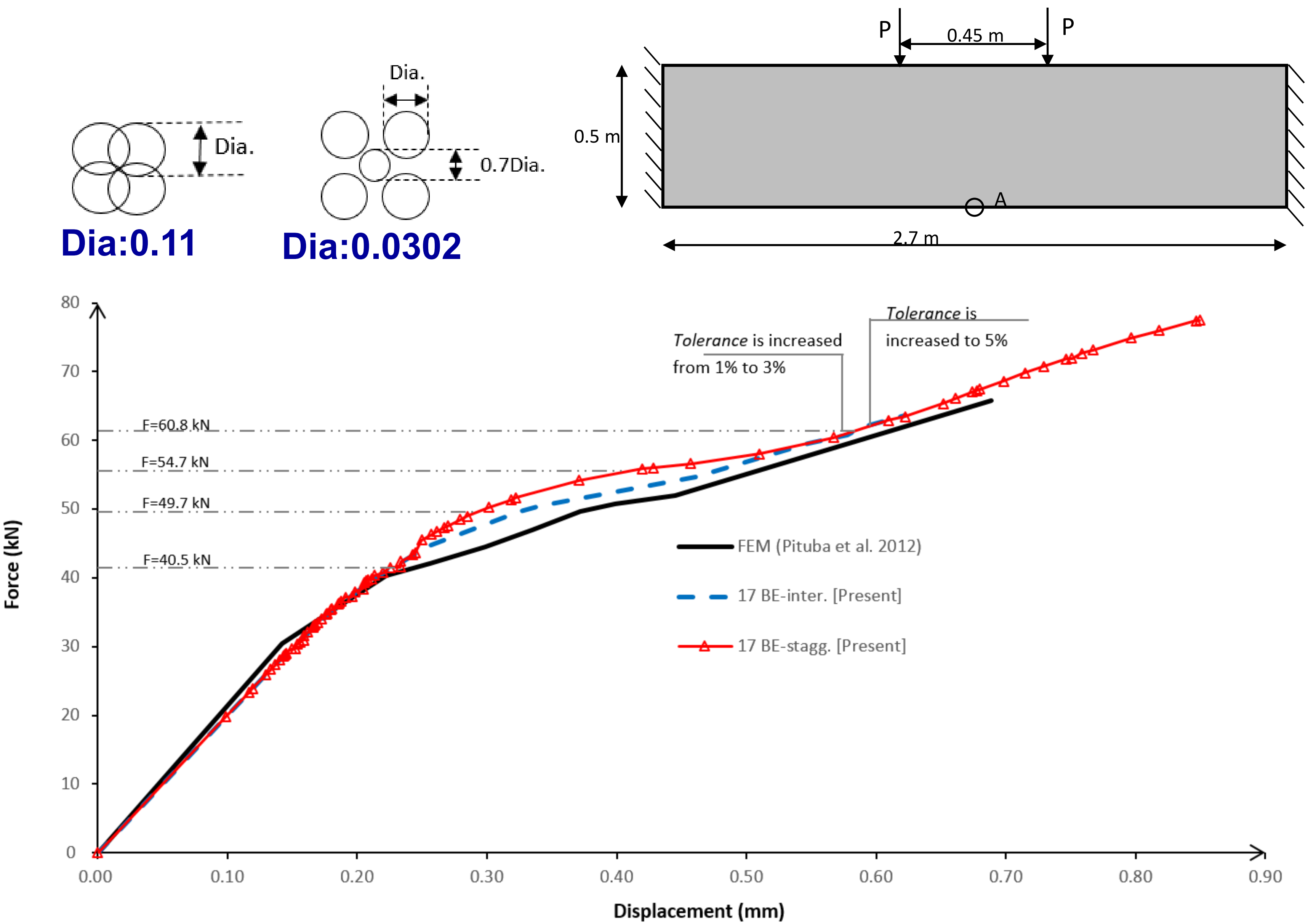


E=247*10^8 N/m^2

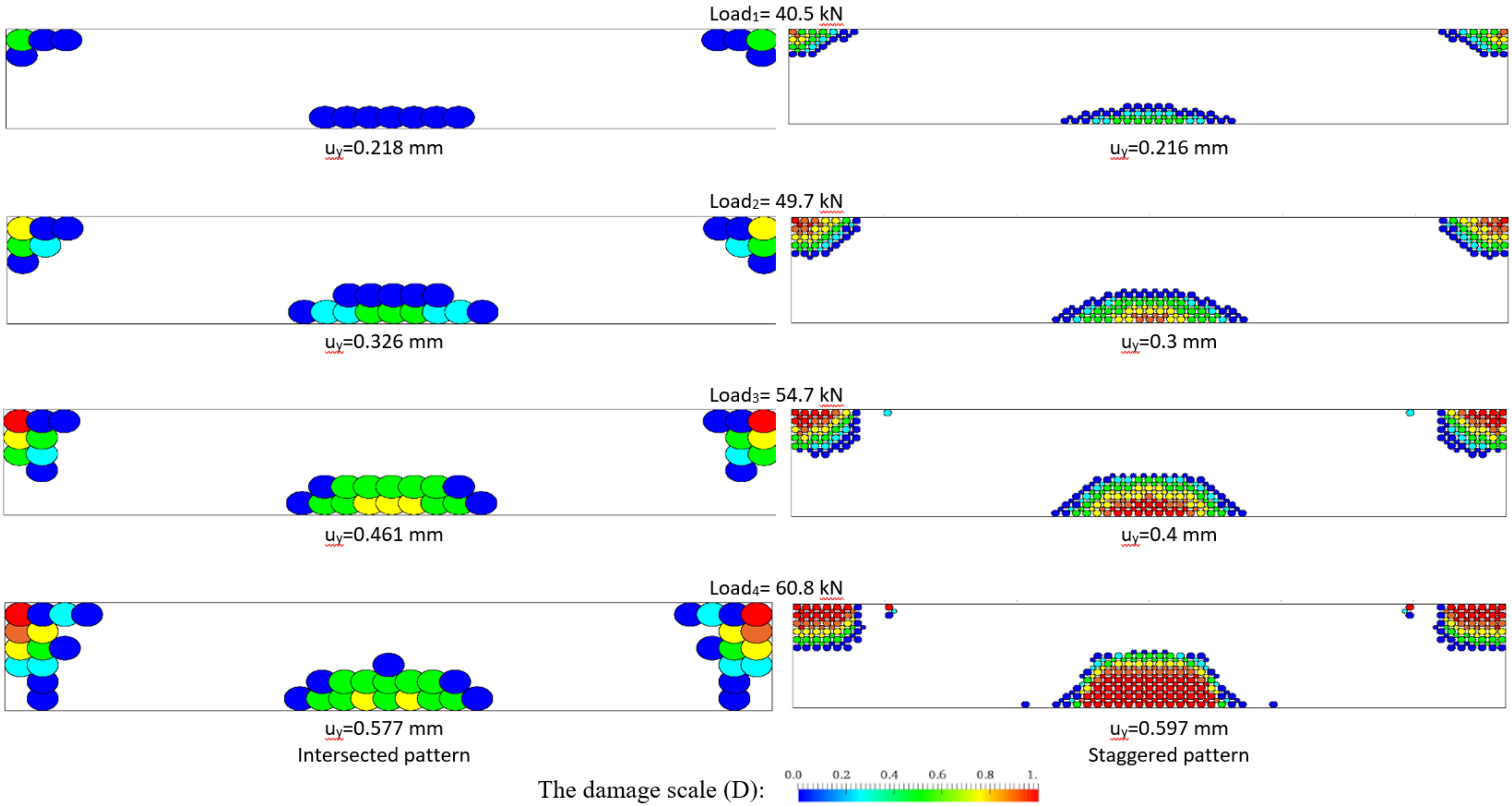
ε_D=0.000067 a=0.7, b=8000

v=0.2 width 0.2 m

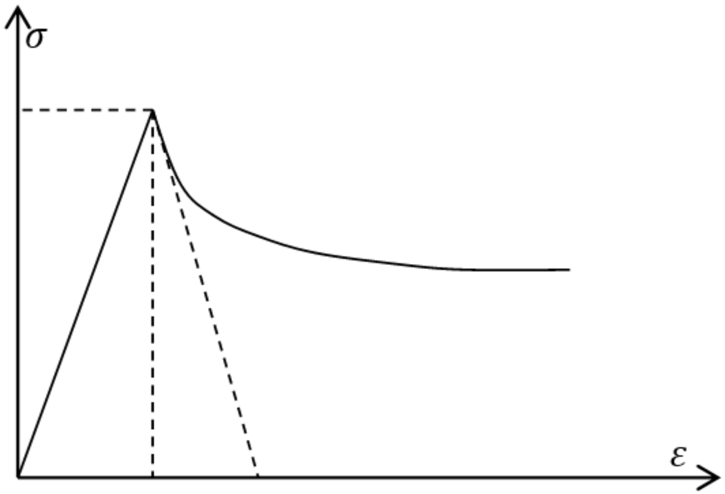
$$D(\varepsilon^*) = \begin{cases} 1 - \left[\dfrac{\varepsilon_D(1-a)}{\varepsilon^*} + \dfrac{a}{exp\big(b(\varepsilon^* - \varepsilon_D)\big)}\right] & if\ \varepsilon^* \geq \varepsilon_D \\ 0 & if\ \varepsilon^* < \varepsilon_D \end{cases}$$


Dia.
Dia.
0.7Dia.
Dia:0.11
Dia:0.0302
P
0.45 m
P
0.5 m
A
2.7 m
Force (kN)
Displacement (mm)
F=60.8 kN
F=54.7 kN
F=49.7 kN
F=40.5 kN
Tolerance is increased from 1% to 3%
Tolerance is increased to 5%
FEM (Pituba et al. 2012)
17 BE-inter. [Present]
17 BE-stagg. [Present]

Fixed-Fixed Beam(Damage pattern):



Nonlocal damage

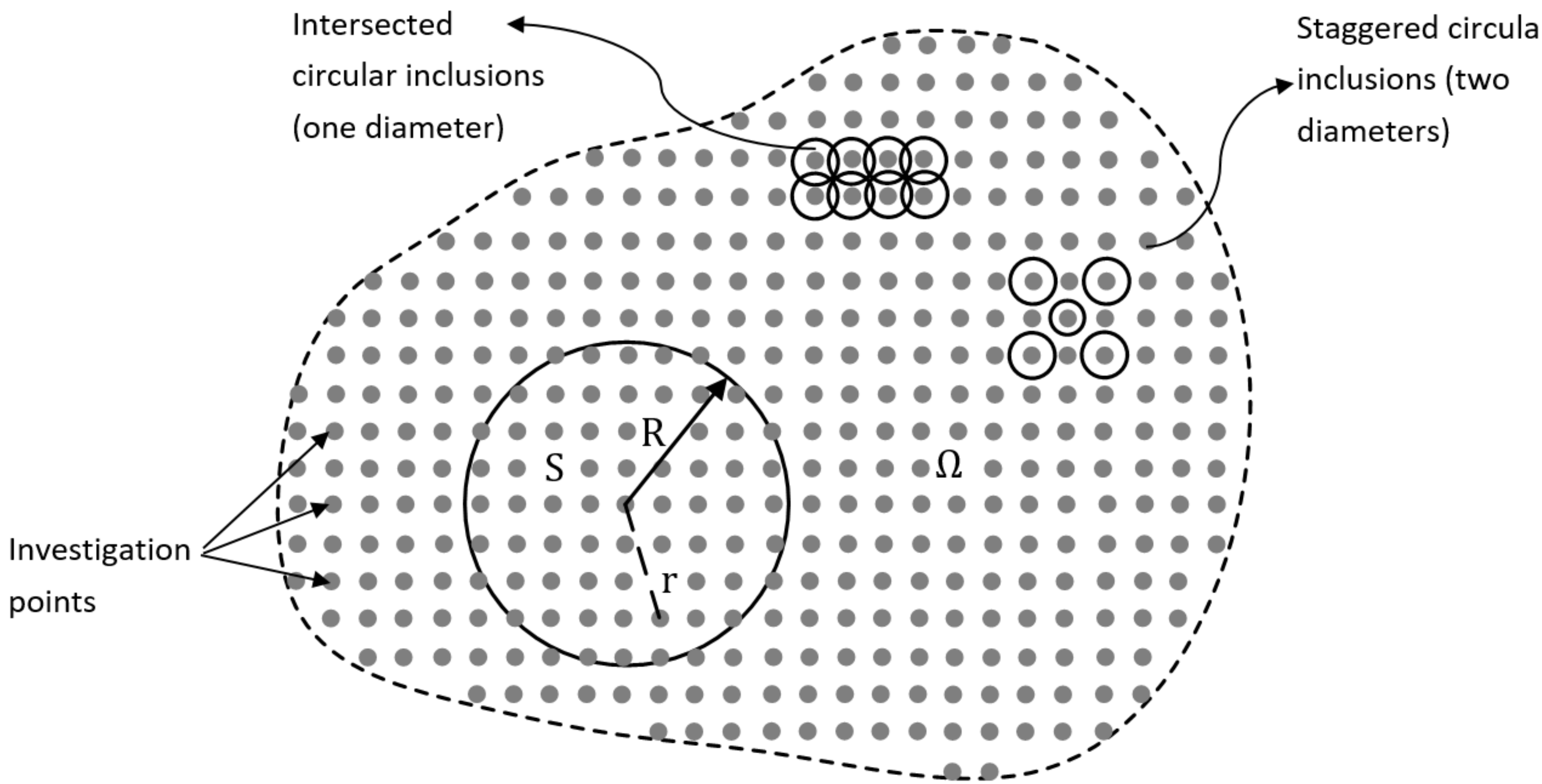


Strain softening

ill-conditioning

Mesh dependence

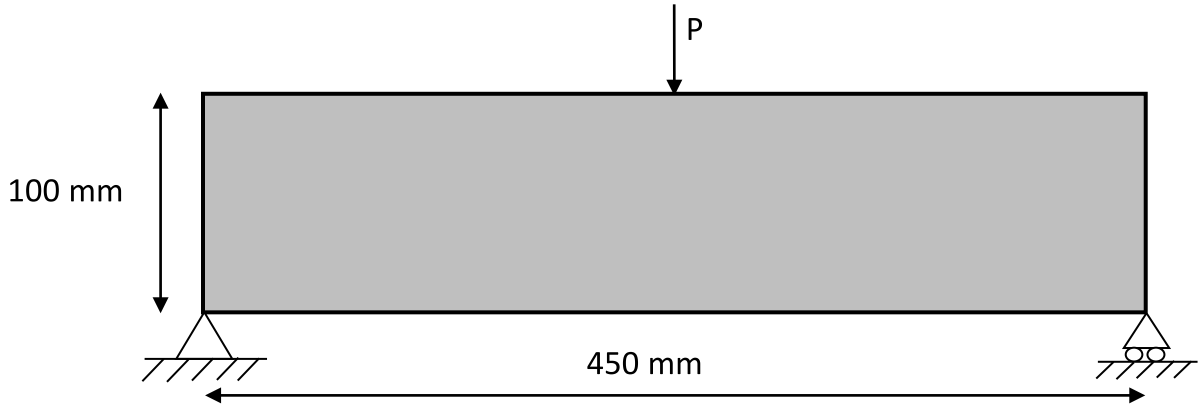
Nonlocal damage



$$f_{nl}(X_p) = \int_S \alpha(r) f(X) dS \Big/ \int_S \alpha(r) dS$$

$$\alpha(r) = \begin{cases} \left(1 - \frac{r^2}{R^2}\right)^2 & 0 \le r \le R \\ 0 & r > R \end{cases}$$

Simply supported beam:



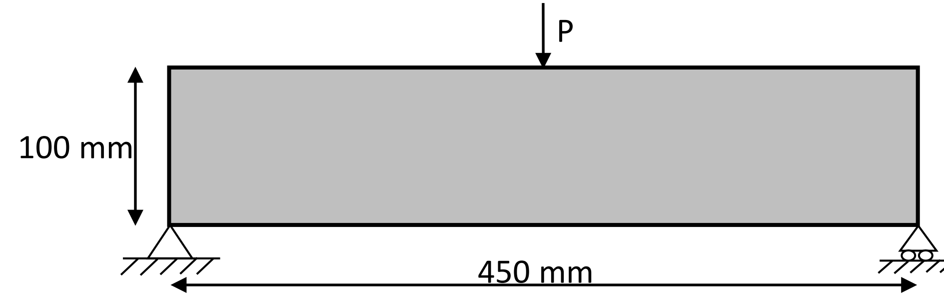
E=21670724658 N/m^2

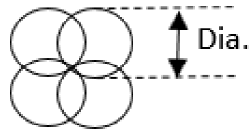
ε_D=0.00009 ε_f=0.005 R=8mm

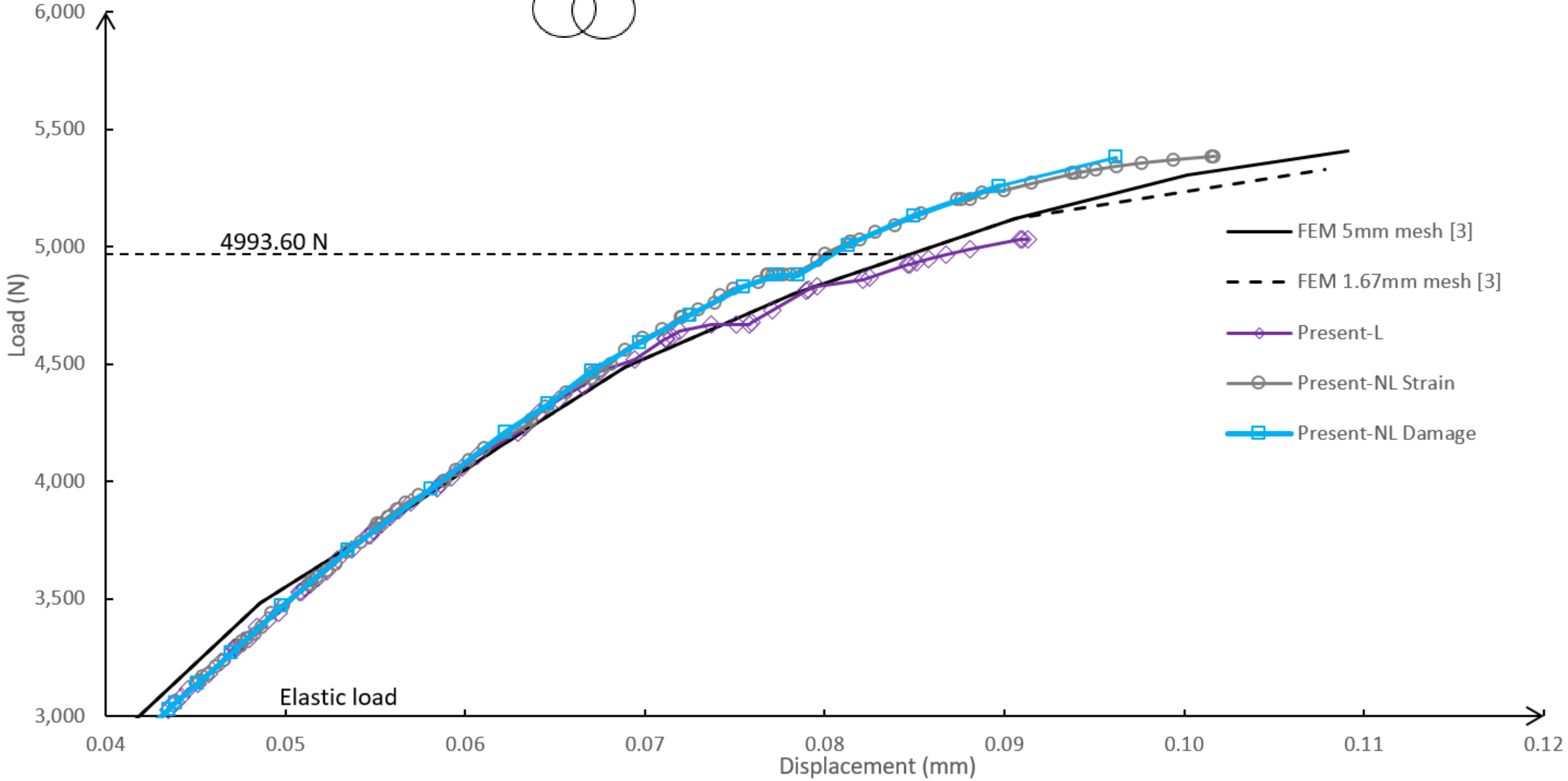
v=0.2 width=100 mm

$$D(\varepsilon^*) = \begin{cases} 1 - \dfrac{\varepsilon_D}{\varepsilon^*} exp\left(-\dfrac{\varepsilon^* - \varepsilon_D}{\varepsilon_f - \varepsilon_D}\right) & if\ \varepsilon^* \geq \varepsilon_D \\ 0 & if\ \varepsilon^* < \varepsilon_D \end{cases}$$

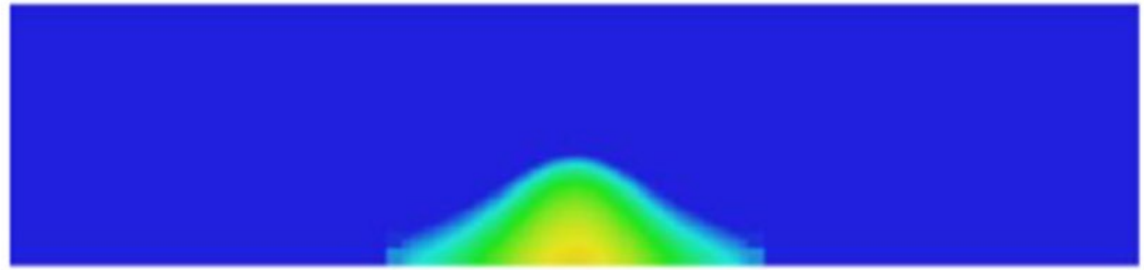
47 BE
Intersected pattern 5 mm



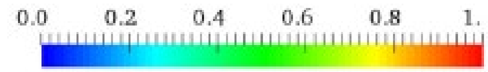


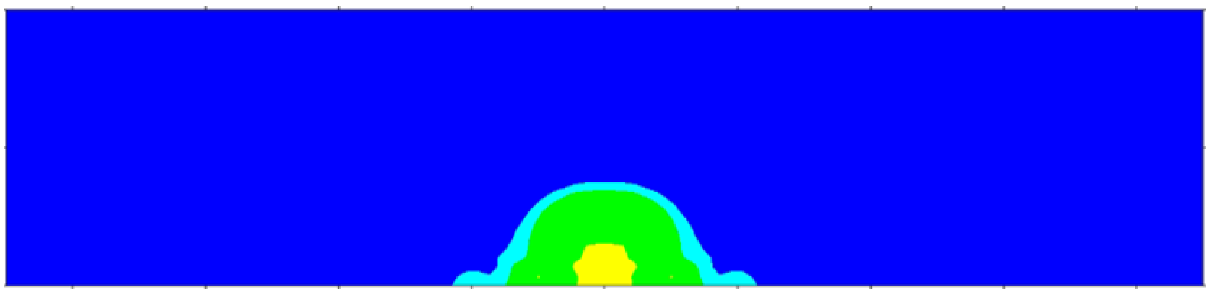


Damage Pattern:

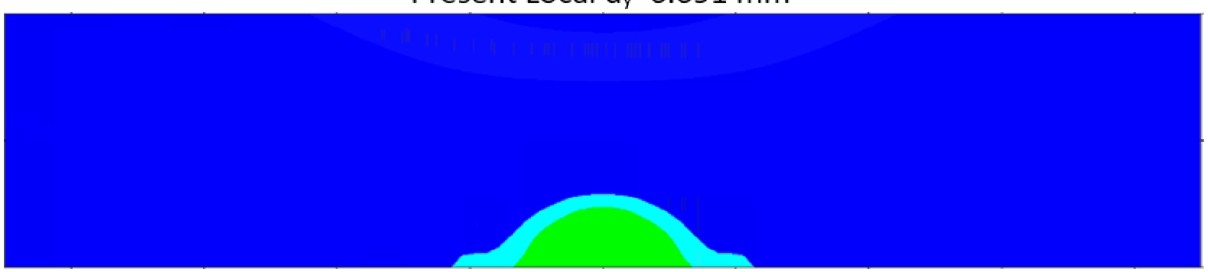


u_y=0.090 mm

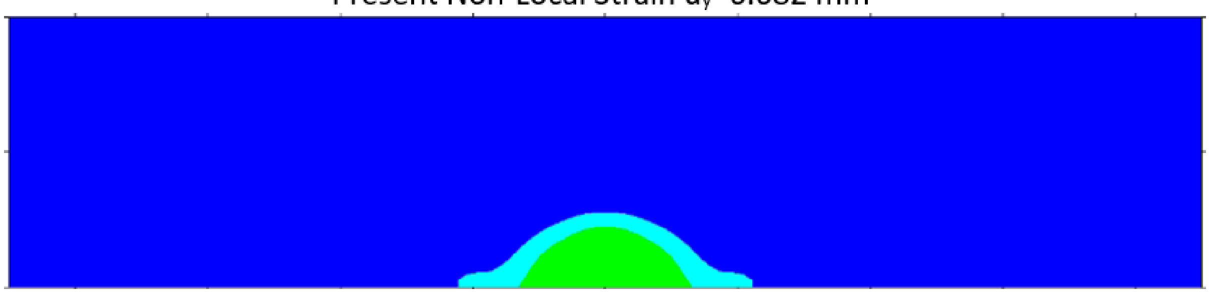




Present Local u_y=0.091 mm

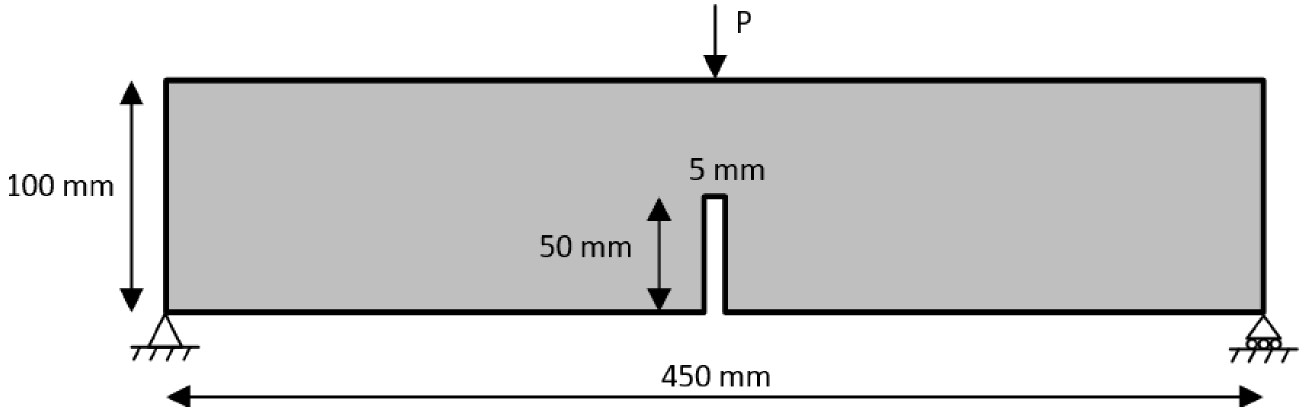


Present Non-Local Strain u_y=0.082 mm



Present Non-Local Damage u_y=0.081 mm

Simply supported beam with notch:

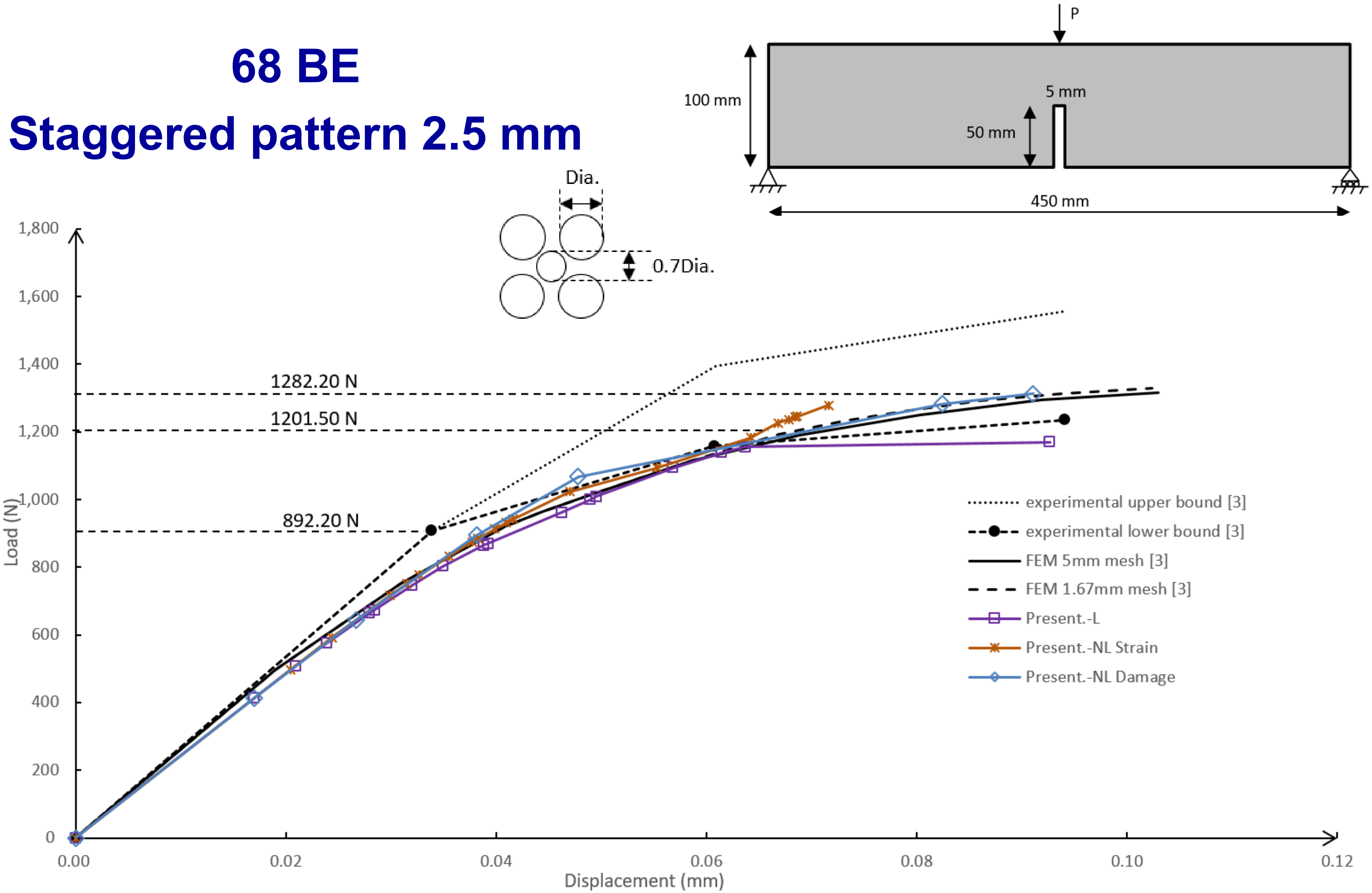


$E=2*10^{10}\ N/m^2$

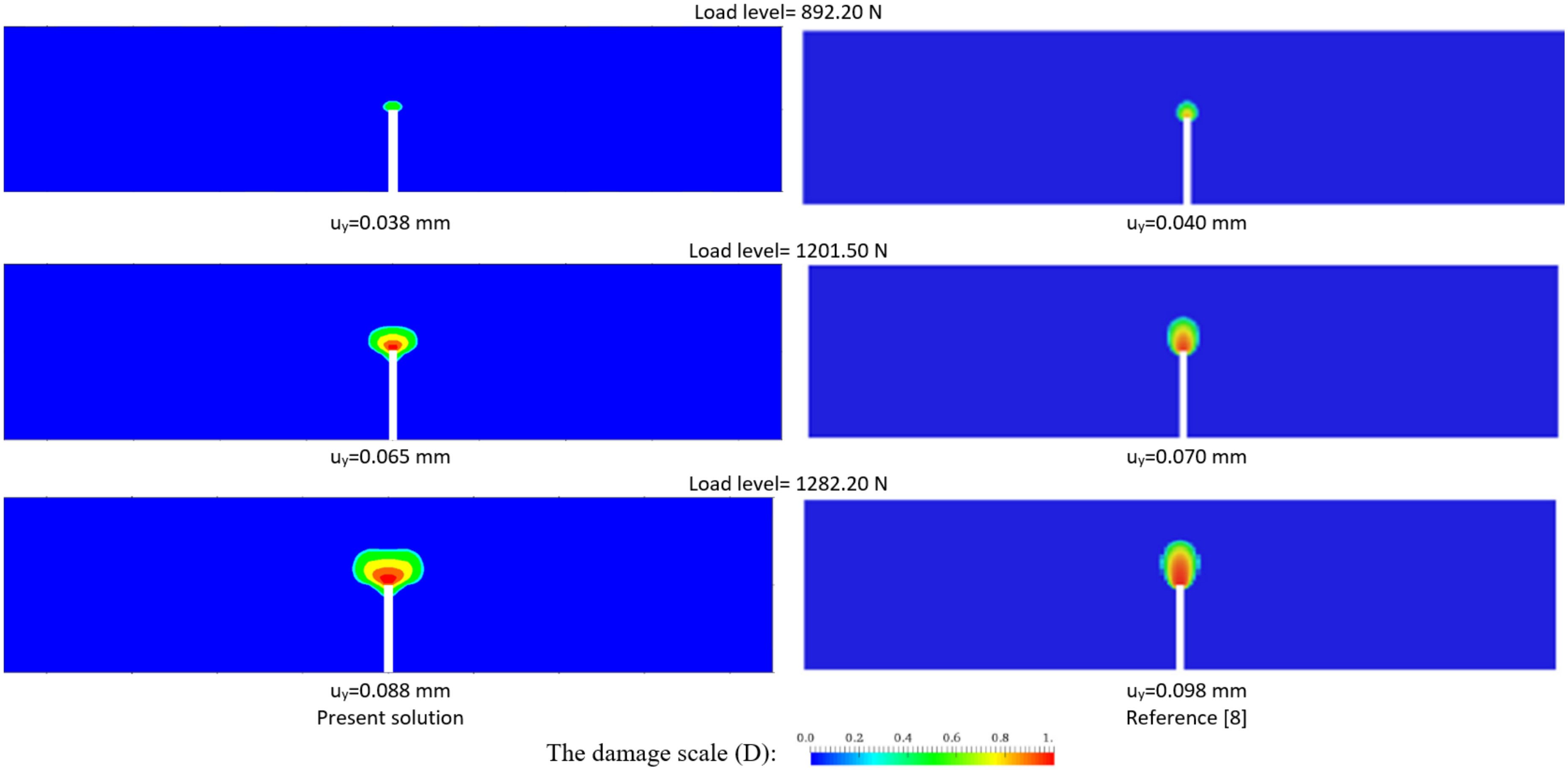
$\varepsilon_D=0.00009$ $\varepsilon_f=0.007$ R=4mm

v=0.2 width=100 mm

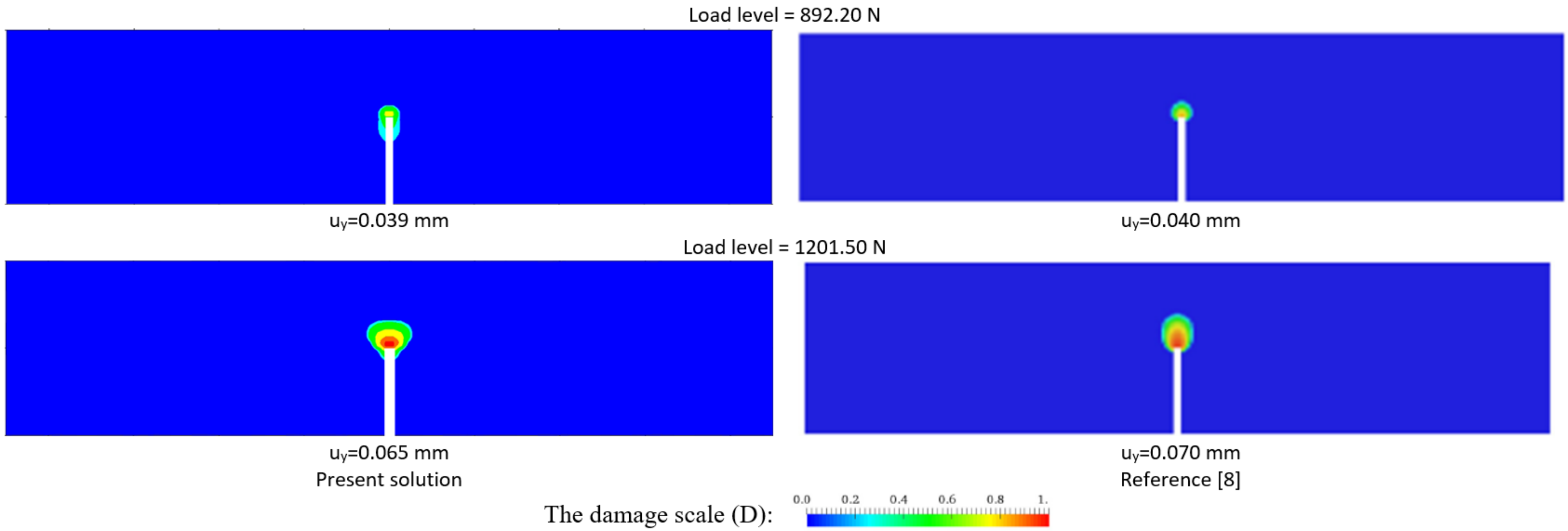
68 BE
Staggered pattern 2.5 mm



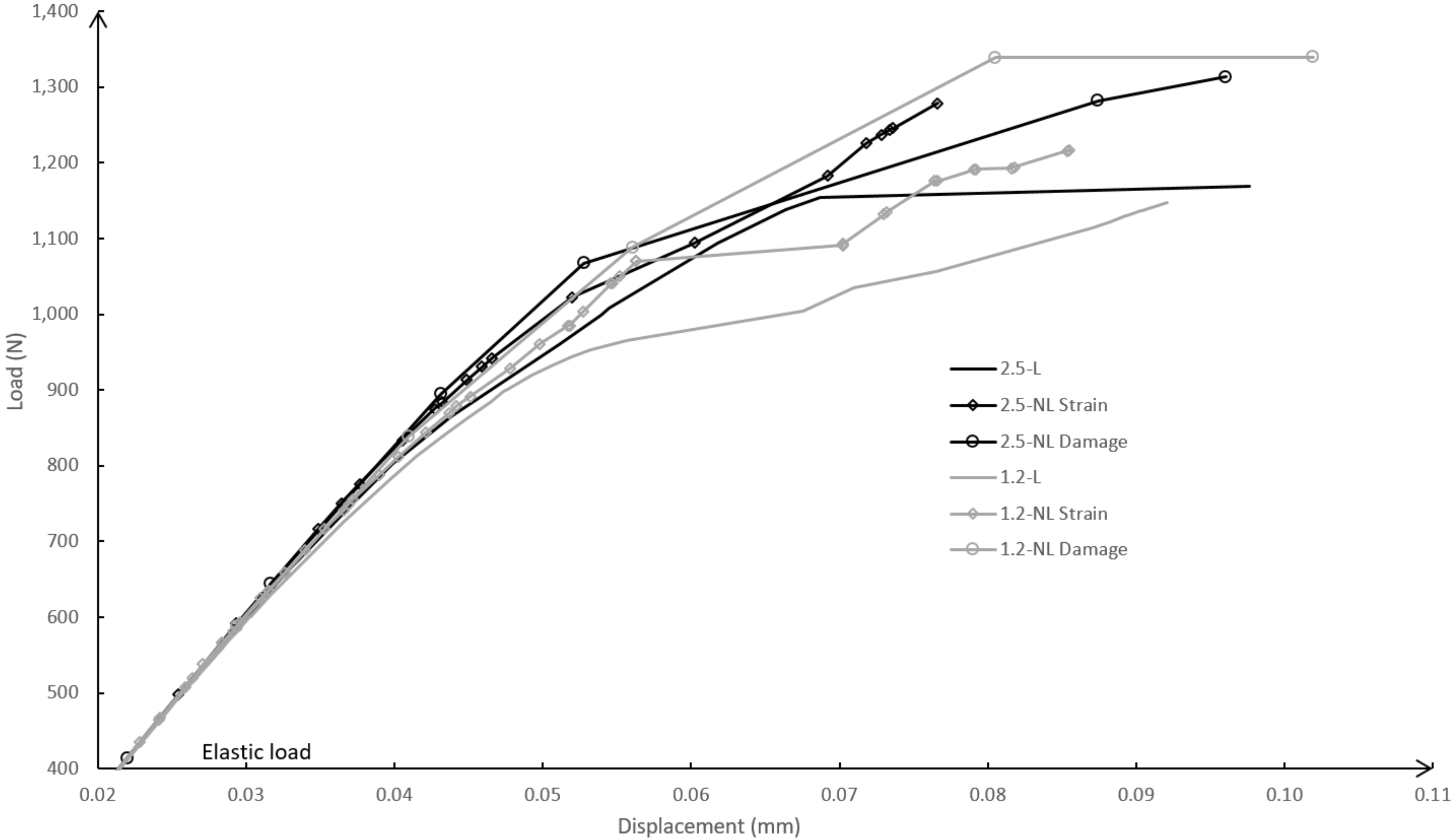
Pattern using damage averaging:



Pattern using strain averaging:



Comparing inclusion diameters:



The End

Thanks for your kind attention